THE USE OF A SAMPLING DEVICE AS AN AID TO TEACHING MANUFACTURING PROCESS CONTROL

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ABSTRACT

The purpose for this research was to develop a manufacturing process control training device which could improve numerous potentially negative characteristics associated with the use of a Sampling Bowl. Among these characteristics were: large size, hindering portability; open containment, risking loss of population elements, potential classroom disruption or detraction from efficiency in classroom experiments; immoderate cycle times between sample extractions, counting and replacement into the general population.

Analysis of existing technology and review of literature, pertaining to process control training devices, resulted in selection of a Sampling Box as a candidate for development. A possible modification to existing designs was conceived; consequently, leading to a more flexible variable, quadropartite sample delivery system.

The resultant Sampling Box is lightweight, compact, highly portable and is hermetically sealed. Flexible sample sizes can be rapidly obtained, counted and repeated by merely depressing one lever. A facilitator's manual was developed to supplement the training device by highlighting many of the tests the Sampling Box is designed to

recapitulate in use. This training aid is now property of the Industrial Education and Technology Department of Western Illinois University.

ACKNOWLEDGEMENTS

This author wishes to extend emphatic appreciation to several individuals whose contributions added greatly to the worthiness of this project. Gratitude is offered to Mr. Jim Warren, President of Lightening Calculator, for his candid interjections pertaining to sampling devices and for his permission to use Figures 2.1 and 2.2.

Thanks are given to Vivienne Schorr (Schorr Consulting) for her evaluation of the Sampling Box and presentation materials. Thanks to Mr. Roy Ervin (Senior Technical Trainer; Sundstrand Aerospace Corporation), for extensive field testing of the prototype model in his process control training classes which has proven the integrity of final product designs and applicability of the Sampling Box to its intended purpose. His insightful comments and suggestions added greatly to the success of this project.

Special gratitude is bequested to Mr. Robert A. Dovich (ASQC Fellow and Quality Assurance Manager of Ingersoll Cutting Tools Corporation) for his developmental advice and contribution of resources for sampling plate generation as well as the use of material from his text Quality Engineering Statistics: ASQC Press. Without his motivation

as friend and mentor, this entire project would have never been undertaken.

CHAPTER ONE

INTRODUCTION

Following a logical progression of mathematical and technological advances, the tools of manufacturing process control were being developed by the mid 1920's. Walter A. Shewhart was the premier advocate of process control methodology with his development of the first control charts for manufacturing purposes. The result of this work can be viewed as a process by which an action takes place based on measurement taken from a manufacturing operation for comparison to an existing standard. His systematic approach toward refinement and maintenance of product quality levels has become the basis for manufacturing process control as it is known to date.

Propagation of these new techniques, although initially sluggish, was advanced by H. F. Dodge, Holbrook Working and Edwin G. Olds. The culmination of their efforts led to the development of process control training programs for both military and civilian industrial personnel during the second world war. Since that time, an increasing appreciation for the logic of controlling manufacturing processes through systematized approaches has resulted in fairly widespread development of training programs for teaching these principles to prospective users throughout most of the industrialized world.

Increased consumerism and global competition for market place survival has further perpetuated the development of manufacturing process control programs. The need for continuously increasing competence levels of industrial practitioners has given rise to a vast proliferation of training programs offered by industrial organizations, training consultants and academic institutions. These programs broadly range from home training courses, seminars and workshops to college and university level course offerings. Increased specialization of educators and their programs now frequently include industry-specific course offerings to all levels and types of industrial personnel who can often learn these concepts at their own convenience.

Academic institutions, professional societies, economic development agencies and other networks are increasingly embracing manufacturing process control embodiment to include such course content as concepts of process variability, tools of data analysis and inference, basic problem solving skills and effective experimentation. Included in the tools and methodology of manufacturing process control training programs are lecture materials, text or workbooks, video cassette tapes, computer demonstrations and the use of devices as aids in teaching these methods to students.

Some common elements to a successful training program include:

1. Emphasis on job related skills.
2. Practical applications of concepts relative to student needs.
3. Minimized lectures with ample demonstrations, classroom exercises and personalized assignments.
4. A problem solving focus more-so than on formula manipulations or memorization.
5. The use of insightful examples which can lead to mastery of precepts through practice.
6. Development of team building skills.
7. Systematic and logical development of course content in modular form.
8. Clearly defined performance goals.
9. Standard measures of student performance evaluations which are predictably applied.

While an instructor may readily espouse the previously defined elements, the aggregate of this constituency may seem to be a labyrinth of imposing requirements for teaching manufacturing process control concepts to groups of students from diverse backgrounds and career goals. However, simultaneous satisfaction of many of these objectives may be achieved with the aid of specifically designed training devices.

Training devices, which can be mere objects or complex mechanisms, have been generated or constructed for the purpose of assisting in the teaching of manufacturing process control principles in classroom settings. Some of these devices have histories dating back to (or prior to) the development of the field of statistics while others are recent innovations; produced to serve a distinct purpose.

Because interest in the development and use of these training devices has attained magnified importance in neoteric process control training programs, it has become the subject of this research to explore the nature of these devices and their use for the purpose of developing such a device for the use of teaching manufacturing process control in the Industrial Education and Technology Department of Western Illinois University.

CHAPTER TWO

LITERATURE REVIEW

While training devices may be simple instruments or even reasonably sophisticated mechanisms, they all serve some degree and type of utility in providing a number of necessary functions which may be critical to more thorough understanding of process control methodology. These devices are generally used to help simulate sampling and experimental manufacturing conditions in a classroom without the need for actual production processes. Many devices are excellent random number generators and help purvey tangible evidence to students about the nature of process variability as it relates to manufacturing operations.

Because training devices have been developed for the purpose of paralleling the behavior of manufacturing operations, instructors can more easily relate experimental results to the student's specific job related skills necessary to control a process. Training devices are particularly useful for providing practical applications of process control methodology in a manner directly relevant to the needs of a student. When the focus of a class is on developing problem solving or team building skills, training devices can bestow satisfactory demonstrations of process behavior and can serve as the vehicle through which classroom exercises and personal assignments are conducted. If an instructor desires clearly defined performance goals with standard measures of student achievement, training devices can be strategically consecrated toward similarity of test results between students in a class and can be maintained from one course offering to the next.

Training devices have been specifically designed to accommodate the user in a wide variety of ways. A partial listing of some broadly conceived yet general applications for the use of training devices is listed below.

1. Basic probability theory.
2. Point estimates and confidence interval determinations.
3. Acceptance sampling.
4. Control charting.
5. Decision making and risk assessment.
6. Tests of hypotheses.
7. Design and analysis of experiments.
8. Distribution-free or nonparametric testing.

This listing is by no means exhaustive however, it does provide some indication of the types of applications training devices have been successfully used to model in typical educational programs. In fact, when the appropriate device is properly applied, nearly any manner of process control methodology can be taught with the aid of training devices. In most practical settings, the limiting bounds of training device capabilities are often the creativity or imagination of the user in identifying the specific applications for modeling.

When considering the selection of a training device, it may be necessary to determine the extent to which the need for instructor expediency and ease of student assimilation of course content factor into the often limited time constraints available to teach manufacturing process control technology. This evaluation may reveal to the instructor a need for a device (or group of devices) which posses the most ideal characteristics desired for the efficient teaching of course-related material. Some of the most common traits or characteristics desired from training devices are given below.

1. Ease of use.
2. Speed of sample selection.
3. Easy sample counting.

4. Efficient replacement of sample back to population (or resetting the device).
5. Maintenance of population characteristics (or minimizing the potential for loss of sample elements).
6. Minimizing sampling bias.
7. Flexibility in sample size selection.
8. Light-weight construction.
9. Portability.
10. Compactness.
11. Rigidity or durability.
12. Minimal potential for classroom disruption.

Each training device is designed for specific types of applications or uses and may not necessarily satisfy all of the criterion listed above. All of the training devices reviewed by this researcher have been purposefully designed and are quite worthy of use for their intended functions. However, it may be necessary to evaluate the inherent merits of each device as well as its intended utility before selection of a specific training aid or group of training aids. These considerations will likely lead to the most optimal selection and use of training devices for teaching manufacturing process control.

The following list includes some of the most commonly used and marketed types of training devices available for teaching manufacturing process control techniques. Moreover, this list is comprised of the types of training devices the researcher has found to have at least some limited potential in modeling discrete probability theory, sampling distributions of discrete random variables or continuous approximations to discrete random variables (for definitions of these terms, please refer to Appendix C).

1. Random Dice.
2. Quincunx.
3. Run Demonstrator.
4. Roman Catapult.
5. Sampling Bowl.
6. Sampling Box.

RANDOM DICE

Since the time of Blaise Pascal and games of chance, dice have been used as a means of displaying the nature of probability theory. Contemporary Random Dice are particularly well suited for the following applications.

1. Basic probability theory.
2. Binomial law.
3. Geometric distribution.
4. General description of the normal distribution based on a frequency distribution.
5. Chi-square tests of significance.
6. Random number generation.

While standard dice may be used, specially designed dice of up to twenty faces, with numbers ranging from negative nine to positive nine, are commonly marketed for use. These dies may also come in a variety of colors to extend their potential for use. Simple die sets may be purchased for as low as five dollars. More elaborate die sets may run as high as seventy-five dollars; including the price of a shaker-cup.

QUINCUNX

A cousin of Charles Darwin, Sir Francis Galton (a well known geneticist), developed the first Quincunx (pronounced quinn-cux) in 1873. His inquiry into the nature of chance events lead to the discovery that sampling distributions, almost irregardless of the nature and type of their origins, formed frequency distributions very similar to what is now known as the normal distribution. Used as a training device ever since, this mechanism enables an instructor to demonstrate the variability of a production process and generate histograms for various sample sizes and for comparison with decision making criterion. Examples of several Quincunx models, offered by Lightening Calculator, are shown in Figure 2.1.

Depending upon the type and complexity of the Quincunx, these devices can graphically display process capability theory and the basis of control chart development. They are also exellent for physically depicting the effects of sample size selection on drawing inference about population characteristics and how proper adjustment of a process can lead to a state of control.

The operating principle of the Quincunx is very similar to that of a pinball machine in that, beads are allowed to fall from the top of the Quincunx through a series of pins, in random manner, to the bottom of a containment (through channels) where the frequency distribution is formed and the beads can be counted. From this, a wide variety of probability assertions can be made about the resulting sampling distributions.

QUINCUNX

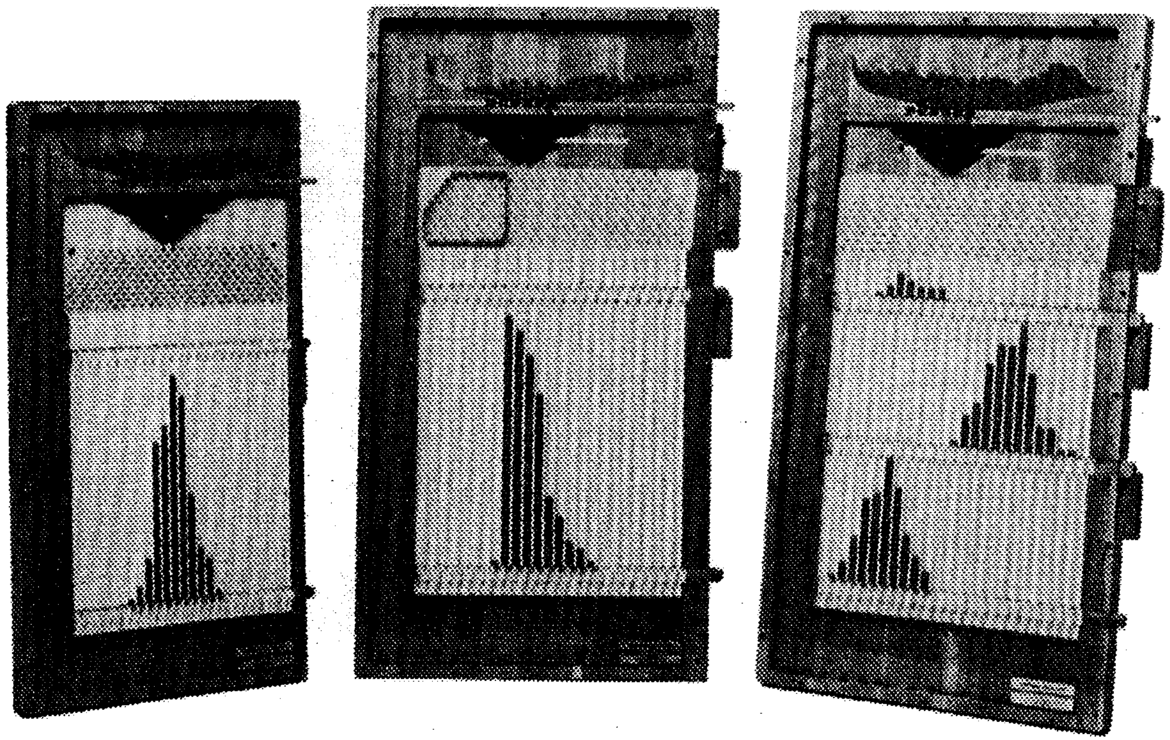

Figure 2.1

Some of the most common modeling applications for the Quincunx are listed below.

1. Description of the normal distribution (binomially based).
2. Point estimates and confidence interval determinations.
3. Control charting ($\bar{x}$ or x, R, precontrol, etc...).
4. Graphical depictions regarding decision making and risk assessments.
5. Physical modeling of tests of hypotheses.
6. Concepts of runs, trends and cycles.
7. Effects of sample size selection on inference sensitivity.
8. Simulating effects of process adjustments.
9. Relation of process spread to product specifications.

Quincunx designs vary widely from one manufacturer to the next. More sophisticated models have adjustable funnels at the top of the Quincunx for demonstrating the effects of adjusting the process center. Multiple pin gates or control slides allow visual depictions of several sampling distributions at a time. Adaptable pin plates are available to show differences of variation between more and less precise manufacturing operations.

Quincunx price ranges vary widely as do their design characteristics. Prices start as low as $250.00 for simpler models and can exceed $1000.00 for the most complex Quincunx configurations. While the purchase prices may seem prohibitive to some users, the greater degree of flexibility offered by more expensive models tends to aid training capability somewhat commensurately. However, practical solutions can generally be purchased between $400.00 and $700.00.

RUN DEMONSTRATOR

Initially designed by Dr. Ellis R. Ott, the Run Demonstrator is an excellent tool for simulating and analyzing the nature of runs, trends and cycles for control charting purposes. While control charts immediately signal out-of-control situations, when points are beyond control limits, they also aid in continued improvement of process performance by detecting the presence of runs, trends and cycles. These conditions indicate a lack of control even though plot points may be within control chart limits.

Run Demonstrators typically consist of a box containing fifty beads; of which twenty-five are red and twenty-five are white. Use of a hand-actuated stop-pin allows one bead at a time to fall from the top reservoir into the lower reservoir. A physical depiction of runs, trends and cycles can then be presented to students for analysis and subsequent probability determinations. Figure 2.2 shows a typical Run Demonstrator. This particular model is offered by Lightening Calculator for commercial sale.

Price ranges for Run Demonstrators generally fall between $150.00 and $225.00, depending primarily on the size and whether the stop pin is spring loaded.

ROMAN CATAPULT

This device has a history as ancient as the Roman empire; dating back several thousand years. Originally used as a weapon for launching projectiles at enemy strongholds, the Roman Catapult has been resurrected for the purpose of teaching process control methodology.

The basic premise behind rehabilitation of the Roman Catapult has been the relative ease by which concepts of the design and analysis of experiments can be authenticated to students. To date, the Roman Catapult is the best device for demonstrating the effects of altering factor types and levels relative to manufacturing processes. By adjusting mast heights, arm lengths, station positions and other possible factors; the Roman Catapult can dramatically portray the criticality of proper factor selection, level setting and the nature of factor interactions.

RUN DEMONSTRATOR

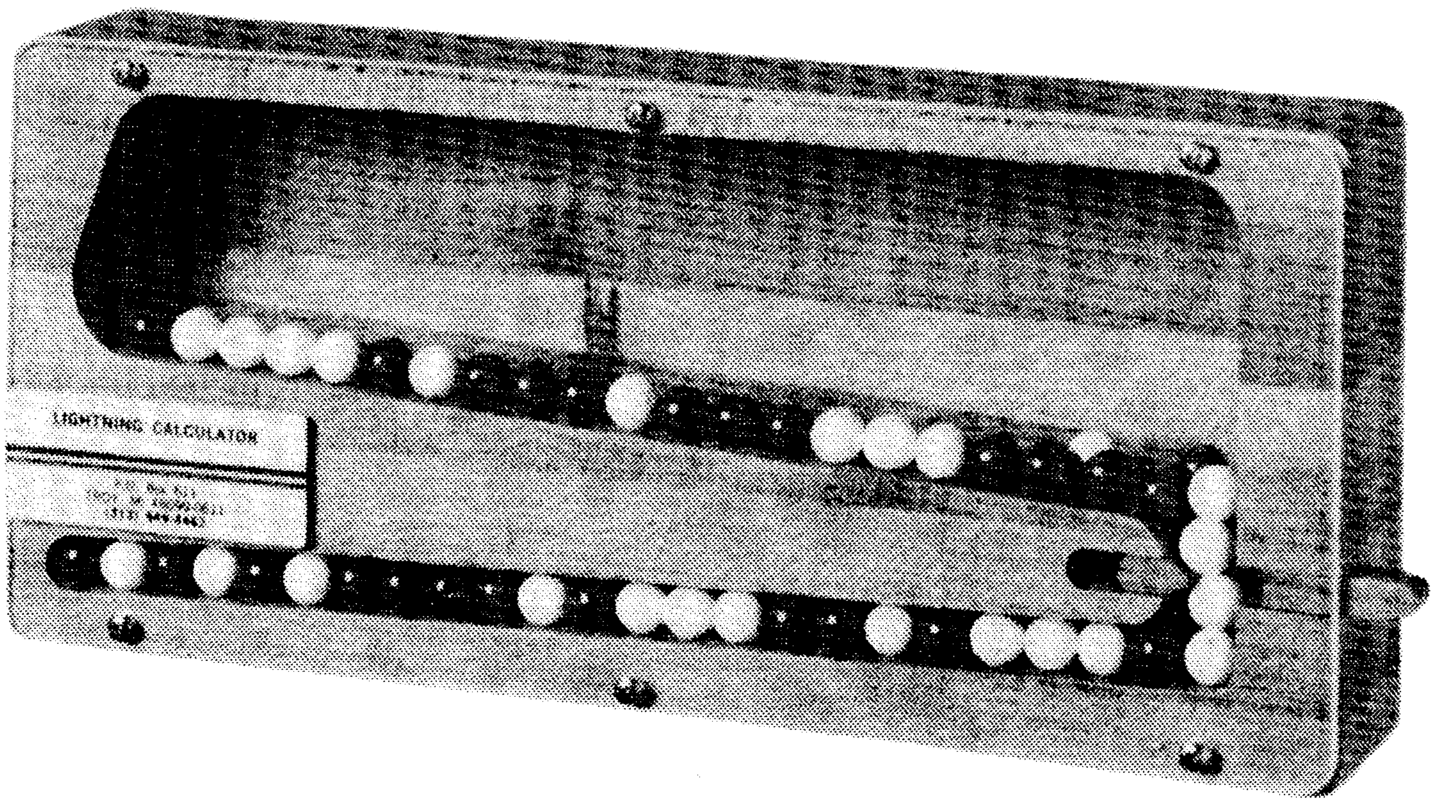

Figure 2.2

Applications of the Roman Catapult to contemporary educational programs are by no means restricted to personations of experimental designs and the generation of test data for analysis. A partial listing of some of its uses is presented below.

1. Point estimates and confidence interval determinations.

2. Control charting.

3. Decision making and risk assessment.

4. Tests of hypotheses.

5. Distribution-free or nonparametric testing.

6. Design and analysis of experiments.

While this training device can service these needs (and others) it is important to recognize that the Roman Catapult has not been designed to generate large amounts of data. The time required to launch projectiles, measure results and reset catapults, compounded with the potential for classroom disruption, hinders its use for large scale number generation. These caveats are not intended to dissuade the user for the Roman Catapult is still the best known device which serves its utility.

There are several catapult designs which are widely marketed for use. While design variations exist, they are all produced to perform essentially the same functions. Price ranges for Roman Catapults commonly fall between $160.00 and $250.00.

SAMPLING BOWL

This device is based on one of the earliest known uses for statistics; basic probability theory. The concept of determining the probability of obtaining a bead of a specific color has been the basis for many experiments conducted by Walter A. Shewhart, W. Edwards Deming and many other bellwethers of process control stratagems.

Sampling Bowls are very simple devices with a great deal of popularity. Their use is equally as simple whereby, one merely reaches into a bowl filled with a mixture of colored beads and draws a sample using scoop or specially designed paddle. The user than counts the beads (buttons, chips or other items) for analysis and probability determinations.

Sampling bowls are very well suited for simulating the output of manufacturing processes in both qualitative and quantitative measures. These devices are designed to produce large numbers of sample items in nearly the most expeditious manner possible. Some of the most common sampling demonstrations are listed below.

1. Binomial law.

2. Poisson Distribution.

3. Point estimates and confidence interval determinations.

4. Acceptance sampling.

5. Control charting.

6. Decision making and risk assessment.

7. Test of hypotheses.

8. Effects of sample size selection.

9. Effects of repeated sampling.

As with other devices described thus far, this listing gives only a partial indication of the potential applications this training device can serve. Western Illinois University also uses a sampling bowl as an integral part of teaching Manufacturing Process Control, IE&T 345.

These training aids are very easy to develop for use. In fact, most of the sampling bowls this researcher has seen have been generated by the very process control educators who use them. Sampling bowls which are commercially marketed offer varying bowl configurations and typically provide 1000 to 3000 beads consisting of between two to seven different colors in varying percentages. Specially designed paddles are commonly offered to provide quick and easy selection of standard sample sizes.

Shewhart Bowls (devised by Walter A. Shewhart), which are very similar to typical sampling bowls, have numbered chips which are also available in multiple colors to expand the utility of these training aids.

Widely marketed sampling bowls are characteristically rugged and durable for long-term use. Depending upon the number of beads and color combinations, bowl construction and the number of paddles purchased; sampling bowls can commonly be purchased for as little as $175.00 and as much as $400.00 for more sophisticated models.

As previously noted, Western Illinois University uses a Sampling Bowl as an aid to teaching Manufacturing Process Control in IE&T 345. This device is essentially a bowl with colored buttons. The sampling population is comprised colored buttons with each color representing various percentages of the entire population. Students obtain samples by scooping buttons out of the bowl and counting the number of each respective, colored buttons. This count data can then be analyzed for use in drawing inferences about the characteristics of the sampling population (or universe).

The sampling Bowl is a very useful device which can help demonstrate most of the concepts related to manufacturing process control training and provides students with a great deal of information about the nature of process variability in a reasonably simple and tangible way. While this device is of great utility in simulating manufacturing output for many different purposes, it has been conjectured by this researcher, that improvements in the design of the Sampling Bowl could lead toward more efficient training of process control techniques to students through a number of indirect ways.

The existing Sampling Bowl in the IE&T department is a relatively large device which can be somewhat arduously transported from one classroom to the next or from one educational facility to another. Moreover, portability between students in a classroom is virtually inconceivable. In order for students to take samples, they must practicably get up from their seats and go to where the Sampling Bowl is centrally located.

Because the Sampling Bowl is an open container, the potential for losing buttons and altering population parameters (see Appendix C: Definition of Terms) cannot be wholly prevented. For the instructor who wishes to have established standards by which student comparisons or evaluations are compared, this may prove to be a detriment. Students will inevitably drop buttons and decrease the overall efficiency of the class in taking samples by having to locate lost buttons and replace them back into the population. If students fail to promptly return samples to the population (or decide to keep them), population parameters will change commensurately.

It has also been conjectured, by this researcher, that the time required for students to extract, count and replace samples back into the population can be improved. This process may presently take more than one minute, depending upon the efficiency of the sample taker. If there is pressure by other students to use the Sampling Bowl, the student could hasten the counting process and introduce the potential for error.

Being an open container, the Sampling Bowl reveals the entirety of its contents to the students. In actual manufacturing settings this may not be the case where the only knowledge of the process may be that which is obtained from the sample. Whenever the entire population is in open view to a sample taker, a greater degree of sampling bias is possible. An objective of this researcher is to develop a sampling device which will reveal only the contents of a sample after it has been taken and, therefore, minimize the potential for sampling bias.

Another issue being addressed by this researcher is that of classroom disruption. If a student must leave a desk to take a sample, it is possible that any number of easily imagined events may take place which can distract the continuity of the classroom exercise. This researcher has envisioned the use of a device with greater portability from one student to the next without the need for leaving one's seat. This should improve the efficiency of sample taking and minimize the potential for classroom disruption.

In order to optimize each of the previously noted conditions, it is necessary to develop a training device which will be reasonably lightweight, compact and highly portable. This device should be hermetically sealed with a population which remains invisible to the student and reveals itself only through the samples taken from it. It has been desired to create a sampling device for quick sample taking of specific sizes, easy counting and rapid replacement of samples back into the population. It is also intended that this device be easy to use and still provide essentially the same utility that the Sampling Bowl offers with greater efficiency and continuity of flow through classroom exercises.

SAMPLING BOX

The Sampling Box (also known as the Bead Box) is a subsequent development of the concepts relating directly to the Sampling Bowl and is designed to provide precisely the same services as the Sampling Bowl. The primary difference between the two devices is in their design configurations. The Sampling Bowl is essentially an open container (some have lids) into which the population is placed and samples extracted; whereas, the Sampling Box is hermetically sealed with the population contained within it.

Samples are taken from the Sampling Box by laying the box down on its face and depressing a lever. This locks the sample in place for easy viewing when the box is placed on its back. Once the sample beads have been counted, they are quickly and easily released back into the general population.

The Sampling Box was originally designed for the purpose of providing educators with a device which had superior portability in transporting from one point to the next. This increased mobility also enables students to pass the device back and forth and hence, improves the efficiency of sample taking.

Owing to the greatest similarity of utility to that of the Sampling Bowl, this researcher has chosen the Sampling Box as the candidate for development and presentation to the Industrial Education and Technology department of Western Illinois University. This product possesses the best potential for satisfying the desired characteristics outlined in previous sections. Because a primary goal of this research will be to develop such a device, more will be presented on its merits in a subsequent chapter.

Several examples of Sampling Box designs for commercial use have been identified by this researcher. Contingent upon design characteristics and marketing origins, the Sampling Box can be purchased for between $150.00 and $550.00.

CHAPTER THREE

DEVELOPMENT OF THE SAMPLING BOX

The decision to develop a Sampling Box may appear to be logical, based on the information presented in the last chapter however, the IE&T Department of Western Illinois University already has a Sampling Bowl which is fairly well suited to the needs of both the instructor and his students. Another choice may have resulted in producing utility for services not currently being met by the existing device and would certainly have rounded out the armory of resources available for use by the University. The basic premise behind the development of a Sampling Box is the fact that the current utility exists and is well established. Generation of a product possessing different utility could possibly disrupt the continuity of the educational process of Manufacturing Process Control, IE&T 345 which is not consistent with the ambitions of this researcher.

Development of the Sampling Box involved a certain dichotomous relationship between rational or logically structured processes and unstructured mental conceptualizations associated with these processes. This relationship was somewhat pervasive throughout the entire process leading to the generation of the Sampling Box. For example; the idea that a more compact, highly portable and hermetically sealed device could be created was envisioned by this researcher. This notion was not borne out by some form of configurational synthesis as much as it was by a certain social-utilitarian motivation.

The first step in the process of innovation (however loosely the term may be held) was to define user requirements for a sampling device based on the existing needs of the instructor and his students. Early in the decision making process, it was apparent that improvements in existing utility were possible for this researcher had seen numerous sampling bowls before. The objective was to assure that selection of a training device matched the operating specifications of the class.

The next step involved conceptualization of how utility requirements may best be matched by a subsequent product design and, consequently, how a sampling device could actually be configured. This process was further bolstered by the accumulation of information from a wide variety of sources about training devices and how they could be tailored to suit the needs of the University and its students. Only then did it become clear that the Sampling Box would make a good candidate as a prospect for development.

Continued analysis of accumulated information revealed a number of noteworthy facts about Sampling Box designs and provided direction for developing conceptualizations for design alternatives. While many of these have already been addressed in this research project, others which ultimately lead to generation of the final product have not yet been explored. Sampling Box designs containing a hidden population were those which most closely resembled the desires of this researcher. The primary reasons for this were the minimization of the potential for sampling bias and increased mystery of the population contained within the box.

Another observation was that certain devices were more flexible in delivering variable sample sizes than others. During this period of developing various conceptual designs, it became evident that the means by which samples were taken from the population and produced for viewing would require one of the most important of all design characteristics; that of plate designs.

Existing product designs included the use of specially devised plates which allowed samples to fall through hole patterns in one plate to be held in another plate with matching hole patterns (or intentions). This is performed by placing the box over on its face and depressing a lever which matches the respective intention patterns. This allows beads contained within the box to fall through to a clear viewing plate at the surface of the box. The best designs also provided spring- loaded plates which automatically offset the respective intentions when the lever was released. The box could then be placed on its back for counting colored beads without their falling back into the general population.

Variations in currently marketed products include the number of intentions on either one or both sides of the Sampling Box. Flexibility in sample selection involves the number of classes or cells of intentions. For example, sampling boxes with classes of twenty and thirty matching intentions will deliver both a twenty and thirty piece sample. Likewise, sampling boxes with one class of one-hundred matching intentions will deliver a one-hundred piece sample. In the case of the two-class set of matching intentions, the twenty piece sample may be counted with the thirty piece sample simply being ignored or covered. This practice of ignoring certain samples is quite acceptable however, the process of marking off samples to be ignored or masking them is somewhat awkward and takes away from time best spent for other activities.

It was believed that a product design could be generated which would allow for visualization of only the number of items desired; thus, adding the mystery of the population characteristics contained within the Sampling Box. Another reason for selecting only the number of items to be counted can be related directly to acceptance sampling a lot of material produced by a manufacturing process.

If the acceptance criterion for a twenty piece sample is one-or-less defective items and a container is opened to reveal more than twenty pieces, the sampler is likely to reject the lot of material based on additional items found defective in the container; even though the original sample may have yielded no defective items. This practice occurs frequently in manufacturing settings and adds to the potential for selective attainment (stratification) of samples or bias in the reporting of sample findings. Existing Sampling Box design do not fully account for this phenomenon however, it was conceived that a new product design could minimize this potential for stratification or sampling bias.

This scrutinization of existing product configurations exposed a potential avenue for a design modification which could provide the same degree (or greater) of sample size flexibility offered by any Sampling Box known by this researcher. This design modification ultimately became what this researcher refers to as a "variable, quadropartite sample delivery system". This development is important because it resulted in the one recognizable act of innovation associated with the Sampling Box designed by this researcher (please note that this notion is loosely held because it has not yet been proven through the patent process).

The primary difference between widely marketed sampling boxes and the device created by this researcher is found in the plate designs. Contemporary sampling boxes have plate designs with hole patterns which match when the lever is depressed and offset when the lever is in the standard position. In these cases, a sample is either delivered or not delivered.

The variable, quadropartite sample delivery plate design provides a typical stationary plate near the surface of the box and an adjustable plate directly below the stationary plate. The respective plates each have four classes of twenty-five hole intentions. Flexibility in sample delivery is possible because the adjustable plates have four classes of hole intentions which are each offset from the preceding class by a predetermined amount (1/16").

By partially depressing the lever, one class of samples fills the hole intentions. Depressing the lever further allows the next class of hole intentions to be filled. This process can be continued until all four classes of hole intentions are filled with samples taken from the general population contained within the box.

The adjustable plate is spring loaded such that merely releasing the lever allows the adjustable plate to rest in standard position, systematically locking the sample in its viewing station. The box can then be placed on its back for convenient viewing of the sample units. Simply depressing the lever again allows the sample to fall freely back into the general population. This process can be rapidly repeated to deliver 25, 50, 75 and 100 piece samples.

Because generation of the variable, quadropartite plate design was the most crucial segment to the success of this project, a great deal of initial emphasis was placed on its development. Having conceptualized the design, it became necessary to determine the feasibility of actually producing these plates. The first development step was to make a reference design. This came to fruition by way of preliminary drawings on vellum drafting paper which were placed upon one another and held to a light. By moving the drawing back and forth, this proved the validity of original conceptualizations and plans for perfectly functional plates were developed.

A feasibility study of the generation of functional plates resulted in CNC Machining two aluminum plates 1/4" thick. This process was greatly facilitated by Mr. Robert A. Dovich (ASQC, Fellow) of Ingersoll Cutting Tools Corp. in Rockford, Illinois. Through him, CADAM designs were transformed into SmartCam machining programs and subsequently placed into the flow of their manufacturing process. After analytical investigations and tests of the finished plates, it was confirmed that feasibility of the plate designs would ultimately result in development of a perfectly functional Sampling Box.

Concurrent with this process was the selection and preparation of the contents to be contained within the box. This developer found through experimentation that readily available rubber and plastic beads lacked the dimensional precision necessary to assure operational compatibility with plate designs. After considerable contemplation and feasibility study, precision-machined, ball-bearings were selected as candidates for use.

The same arduous trek required for selection of ball-bearings was necessary for determining a satisfactory means of coloring them. None of the available paintings, coatings or adhesives studied by this researcher were capable of withstanding the constant abrasion they would be subjected to during use. Consultation with Russel Gruner (CEO, Performance One Plating) and John Gruner (CEO, Manner Plating); both located in Loves Park, Illinois, resulted in yellow zinc dichromate and black oxide plating for colored bearings. These choices represented the best abrasion resistance properties as well as the greatest contrast in color combinations available to this developer.

Development of frame designs followed much the same path from conceptualization through engineering feasibility as did other components of the Sampling Box. While the box itself may seem to appear innocuous, several important considerations required addressing before construction. Beyond serving as a reservoir for housing the bearings, frame designs must necessarily accommodate precise yet fluid, spring-loaded movement of the adjustable plate through channels to systematically support union with stationary plate intentions. Compulsory satisfaction of these requisites mandated the proper selection of material composition intended for prolonged service.

Preliminary designs were based on hardwood construction of box frames. The choice of wood as a material for construction was important for several reasons. Owing to the manufacturing processes at the disposal of this developer, wood was the most logical choice of construction material. Wood was economical to acquire and was abundantly available. More important, however, hardwood is rugged and durable enough for long life use with the aluminum plates and is light weight for the portability characteristics desired by this developer.

Consistent with material composition of the frames, a wood backing was chosen for the Sampling box. Its reasonably impregnable and opaque surface conceal population contents as desired. The port on the back allows for easy access by the instructor to alter population characteristics as may be required.

Plexiglass was chosen as an ideal material for the transparent viewing face. Its succeptability to fracture is much less than glass and its scratch resistance is greater than standard plastic materials. These properties will help preserve translucence and fracture resistance as well as providing protection from blemishes due to bearings resting against the back surface of the viewing face.

For convenience of the users, a spring-loaded, self-stationing feature for the adjustable plate allows easy locking of samples in place for viewing. experimentation with the development model resulted in the use of three tempered-steel springs which are embedded in the wall opposite the lever. Resiliency of the springs should remain reasonably resolute for several thousand cycles of use.

After analytical investigations into the nature and types of possible design configurations; construction materials, selection of manufacturing processes, preliminary drawings and engineering specifications were prepared for construction of a development model. These preliminary engineering activities included development of part drawings, material requirements, parts lists and processing methods for manufacturing the box.

Construction of the development model resulted in a fully-functional Sampling Box. With the exception of plate generation, most of the processing was performed directly by (or with the assistance of) this developer. This included frame and cover manufacture, plating of colored bearings, spring assembly and box construction as well as surface finishing of the completed assembly. The ability to take such an active role in performing these functions greatly added to subsequent refinements incorporated into the final product.

The development model was subjected to a wide variety of testing. Among these tests were cyclic testing of the sample delivery system, plating wear, durability of construction and randomness of sample extractions. Some of these tests were fairly extensive and somewhat brutal on the development model. This was necessary because the intention was to develop a training device which could endure harsh use over the life of the finished product.

Generation and testing of the development model proved highly successful in exposing the inherent strengths and weaknesses of preliminary design characteristics. This lead to refinements in structural design and construction methods, reflected by subsequent drawings and specifications. The aggregate of these activities are manifest in what is now the field test model or prototype.

Throughout the entire process of developing the Sampling Box, reasonable adherence to a rational or logical approach toward systematic progression from one step to the next was maintained. From commencement of preliminary activity through completion of the final product, the process of producing the Sampling Box consumed approximately three months. This was followed by nearly four months of continuous testing and use by Mr. Roy Ervin (Senior Technical Trainer; Sundstrand Aerospace Corporation) in his daily process control training sessions. Only now can this developer confidently transfer ownership of the Sampling Box over to the Industrial Education and Technology Department of Western Illinois University.

CHAPTER FOUR

FACILITATOR'S APPLICATION MANUAL

THE NORMAL DISTRIBUTION

Although the purpose for this research is primarily to describe attribute characteristics using discrete probability distributions, it is necessary to first explore the nature of variability and the assumption of normality. The concept of normality will first be introduced through a description of the normal distribution; not merely because the assumption of normality is conveniently described but rather, that the normal distribution is often used to approximate discrete probability distributions in experimental situations. In fact, "A random variable that is an average or a sum of values of another random variable is, under very general conditions, almost always distributed approximately as a normal random variable, regardless of the form of the distribution of the random variable with values that are summed or averaged."[1]

The normal distribution is one of the most important and widely used of all the probability distributions. The random variable of a

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[1]Roger C.Pfaffenberger & James H.Patterson, Statistical Methods (Homewood, IL: Richard D.Irwin,Inc.,1987), p.272.

normal distribution is continuous by nature such that the population variable can assume any possible value along a number line within a given probability or confidence range. This is in marked contrast with discrete random variables which have finite population values (yes, no, good, bad, etc...). This point will become evident in later sections.

The normal distribution has some important characteristics which make it useful for estimating product or process quality. It has a distinctive unimodal, bell-shaped curve which is symmetrical about a mean or process average. Because the random variable associated with the normal curve can posses any one of an infinite number of possible values, the probability of obtaining any one specific value is actually zero. However, the probabilities can be measured by the area between specific values. The area under a normal curve represents the sum of all possible probabilities between negative infinity and positive infinity and equals one. The area or cumulative density between values becomes the basis for making probability assertions about sampling populations.

If a random sample is taken from a population which is normally distributed, the population mean (μ) can be estimated and is given by:

$$\mu \approx \underline{X} = \frac{\sum_{i=1}^{n} (x_i)}{n} \qquad (4.1)$$

where: $\sum_{i=1}^{n}$ = the summation of values (x_i) taken during sampling

$\bar{X}$ = an estimate of the population mean

n = the sample size

x_i = the ith value for the random variable X

Example. Using Equation 4.1 and the data in Table 4.1, find the estimate value $(\bar{X})$ for the population mean (μ).

$$\sum_{i=1}^{n} (x_i) = 125$$

$$n = 25$$

$$\mu \approx \bar{X} = \frac{\sum_{i=1}^{n} (x_i)}{n} = \frac{125}{25} \text{ or } 5$$

This estimated value $(\bar{X})$ for the population mean (μ) is five.

The variability of a random variable (X) for the normal distribution can be measured by the variance (σ^2). Estimates of the population variance (σ^2) are given as s^2 and can be determined by :

SAMPLE DATA FOR CALCULATION OF POPULATION MEAN

5	7	6	3	4
6	4	3	4	7
7	4	5	7	6
3	6	4	6	5
4	5	6	4	4

Table 4.1

$$\sigma^2 \approx \underline{s}^2 = \sum_{\underline{i}=1}^{\underline{n}} \frac{(\underline{x}_{\underline{i}} - \overline{\underline{X}})^2}{\underline{n}-1} \tag{4.2}$$

where: $\overline{\underline{X}}$ = the sample average
$\underline{n}$ = the sample size

For the convenience of the reader, Table 4.2 shows the calculation of the sample variance ($\underline{s}^2$) using the data from Table 4.1 and Equation 4.2. The value ($\underline{s}^2$ = 1.75) calculated in this example represents an estimate of the true population variance (σ^2) and depicts the variability of this population.

Measurement of variability for the normal distribution is generally represented by the standard deviation (σ) and is the square root of the variance (σ^2) as given by:

$$\sigma \approx \underline{s}^2 = \sqrt{\underline{s}^2} \tag{4.3}$$

Example. Using equation 4.3, a calculation of the estimate of the standard deviation associated with the last example is given below : $\underline{s} = \sqrt{1.75} = 1.3229+$.

CALCULATION OF POPULATION VARIANCE

x_i	$(x_i - \bar{X})$	$(x_i - \bar{X})^2$
5	5-5 = 0	0
6	6-5 = 1	1
7	7-5 = 2	4
3	3-5 = -2	4
4	4-5 = -1	1
7	7-5 = 2	4
4	4-5 = -1	1
4	4-5 = -1	1
6	6-5 = 1	1
5	5-5 = 0	0
6	6-5 = 1	1
3	3-5 = -2	4
5	5-5 = 0	0
4	4-5 = -1	1
6	6-5 = 1	1
3	3-5 = -2	4
4	4-5 = -1	1
7	7-5 = 2	4
6	6-5 = 1	1
4	4-5 = -1	1
4	4-5 = -1	1
7	7-5 = 2	4
6	6-5 = 1	1
5	5-5 = 0	0
4	4-5 = -1	1

$$\sum (x_i - \bar{X})^2 = 42$$

$\bar{X} = 5$; $n = 25$

$$\sigma^2 \approx s^2 = \frac{\sum_{i-1}^{n} (x_i - \bar{X})^2}{n - 1} = \frac{42}{24} = 1.75$$

Table 4.2

If a continuous random variable ($\underline{X}$) is normally distributed, the shape of the distribution and the probabilities associated with the random variable are dependent upon the probability density function (pdf) which is given as:

$$f(\underline{y}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\underline{x}-\mu)^2}{2\sigma^2}} \tag{4.4}$$

where: $\underline{y}$ = the corresponding ordinate for the absissa value of $\underline{x}$
e = a commonly used mathematical constant (2.718+)
π = a commonly used mathematical constant (3.141+)
μ = the population mean
σ = the population standard deviation

Example. Consider the data used in the last two examples. Assume that the values of the sample mean ($\overline{\underline{X}}$) and the estimate of the standard deviation ($\underline{s}$ = 1.3229) are the true population parameters (see Appendix C; Definitions of Terms). Use Equation 4.4 to determine the corresponding ordinate for the absissa value of $\underline{x}_i$ when $\underline{x}_i$ equals four.

$$f(4) = \frac{1}{(1.3229)\sqrt{2\pi}} e^{-\frac{(4-5)^2}{(2)(1.3229)^2}} = .2266+$$

Thus, the corresponding ordinate for the given value of $\underline{x}_i$ (4) is approximately equal to $\underline{y}$ (.2266). This value is represented in Figure 4.1 as the circled point. Figure 4.1 depicts the probability density function (pdf) for the values given in the previous examples and is the normal curve associated with the determined population parameters ($\underline{\overline{X}}$ = 5; σ = 1.3229). Please note that these assumed population

parameters are (in reality) only estimates of these values; presented for demonstration purposes. For a treatment of estimated values, the interested reader is encouraged to review the text by Dovich cited in the Reference section at the end of this presentation.

For the convenience of the reader, Figure 4.2 has been generated to show the area (cumulative probability) contained within plus-and-minus one, two and three standard deviations from the population mean (μ).

The areas presented in Figure 4.2 are readily determined when the normal distribution is used in a standardized form with a mean (μ) equal to zero and a variance equal to one. The purpose for standardization is to provide for computational simplicity and a recognizable format for general use.

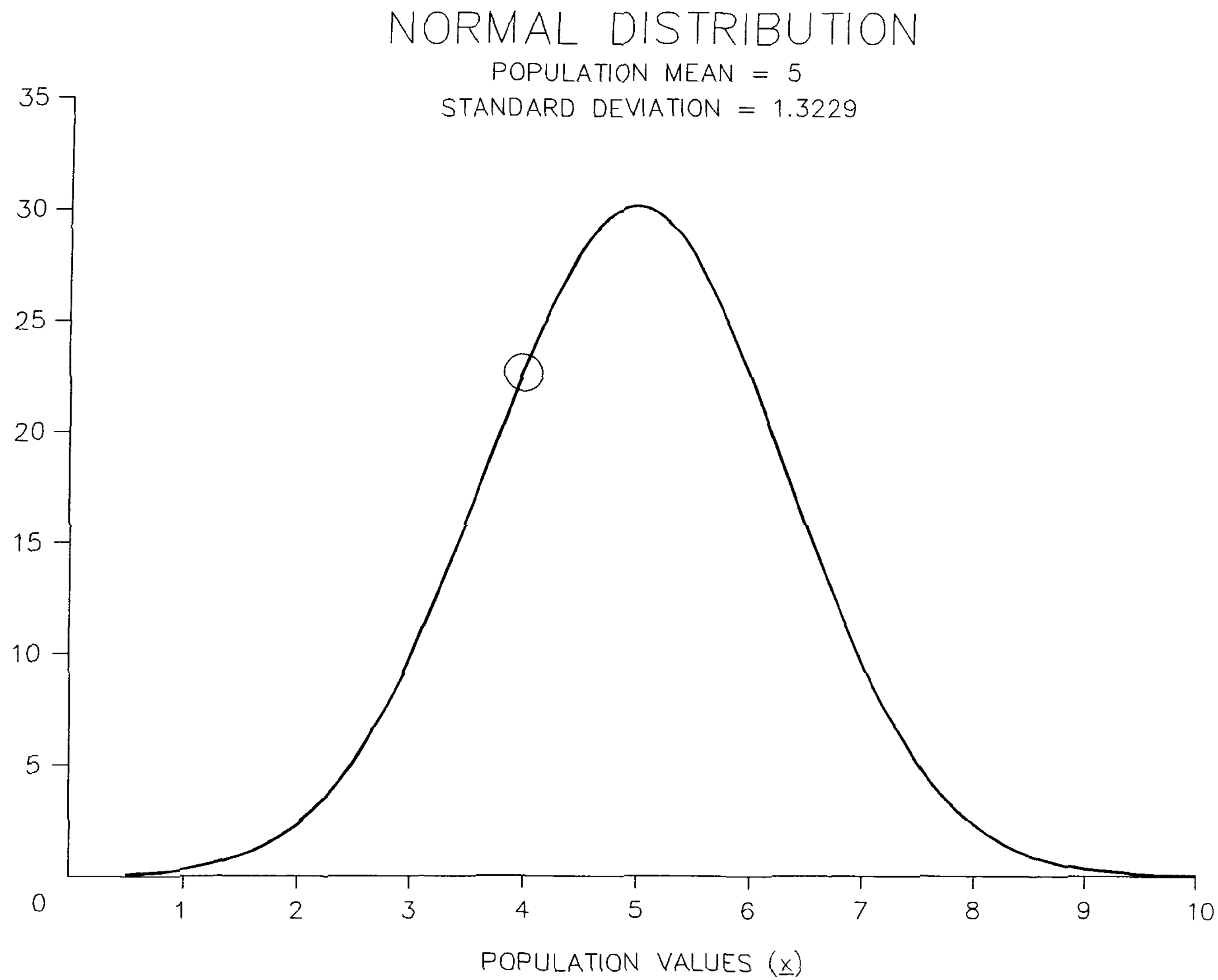

NORMAL DISTRIBUTION
POPULATION MEAN = 5
STANDARD DEVIATION = 1.3229
y=f(x) X 100
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
POPULATION VALUES (x)

FIGURE 4.1

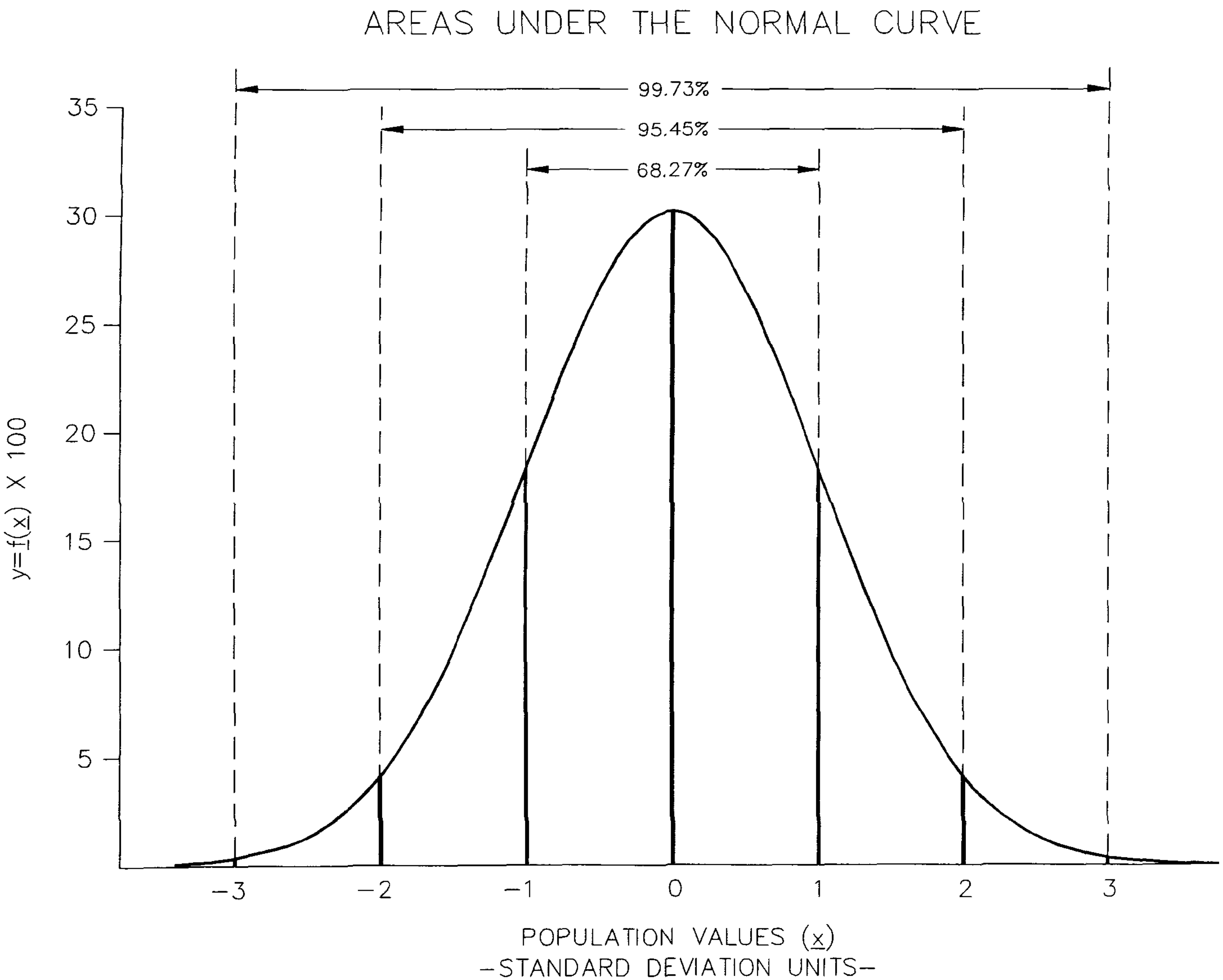

AREAS UNDER THE NORMAL CURVE
99.73%
95.45%
68.27%
y=f(x) X 100
35
30
25
20
15
10
5
−3
−2
−1
0
1
2
3
POPULATION VALUES (x)
−STANDARD DEVIATION UNITS−

FIGURE 4.2

When the normal distribution has been standardized, the use of specifically devised tables and the following formula can yield probability determinations.

$$\underline{Z} = \frac{x_i - \mu}{\sigma} \qquad (4.5)$$

where: $\underline{Z}$ = a standardized test statistic value, in standard deviation units, for comparison with expected tabular values

Example. Using equation 4.5 and the data below, calculate the value for $\underline{Z}$. Where :$\mu = 5$; $\sigma = 1.3229$; $\underline{x}_i = 7.593$. Substituting these values into Equation 4.5 yields the following result :

$$\underline{Z} = \frac{7.593-5}{1.3229} = 1.96$$

The calculated value of $\underline{Z}$ ($\underline{Z}$-score) for this data is 1.96.

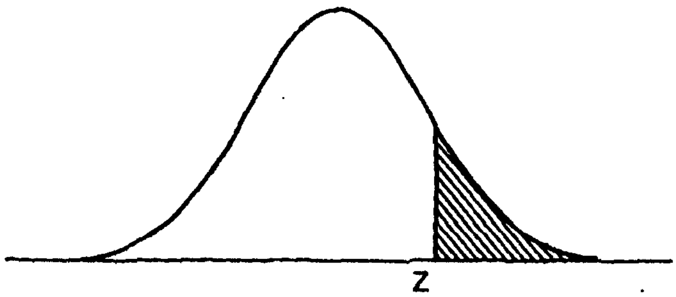
Example. Using Equation 4.5 and Table 4.3 for the data presented below, determine the probability that the random variable $\underline{X}$ falls between these two values. Where: $\underline{x}_1 = 2.407$; $\underline{x}_2 = 7.593$; $\mu = 5$; $\sigma = 1.3229$. Find $\underline{P}(x_1 \leq \underline{Z} \leq x_2)$ or $\underline{P}(2.407 \leq \underline{Z} \leq 7.593)$.

$$\underline{Z}_{\underline{x}_1} = \frac{2.407-5}{1.3229} = -1.96$$

$$\underline{Z}_{\underline{x}_2} = \frac{7.593-5}{1.3229} = 1.96$$

Normal Distribution

AREA BEYOND Z



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4	.3446	.3481	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

Table 4.3

$\underline{P}(2.407 \leq \underline{Z} \leq 7.593) = 1- (.0250 + .0250) = .95$ or 95%.

This states that ninety-five percent of all values from a normal distribution, with these parameters, will fall between these two values. Figure 4.3 graphically depicts this relationship as the area within the confidence interval bounded by the two lines.

As previously stated, the development of the concepts presented in this section are a necessary prelude to the concepts which will follow as they will often be used as a basis for predicting probabilities in subsequent sections.

POINT ESTIMATE FOR EXPECTED VALUE MEAN OF A PROPORTION (p)

For attribute data, taken from a discrete probability distribution, the average proportion (p) or expected value for occurrences from a population can be estimated by the following equation:

$$\underline{p} = \frac{\text{number of occurances}}{\text{sample size}} \tag{4.6}$$

An example of the use of Equation 4.6 can be illustrated by analyzing the contents of a sampling box which was built for use by

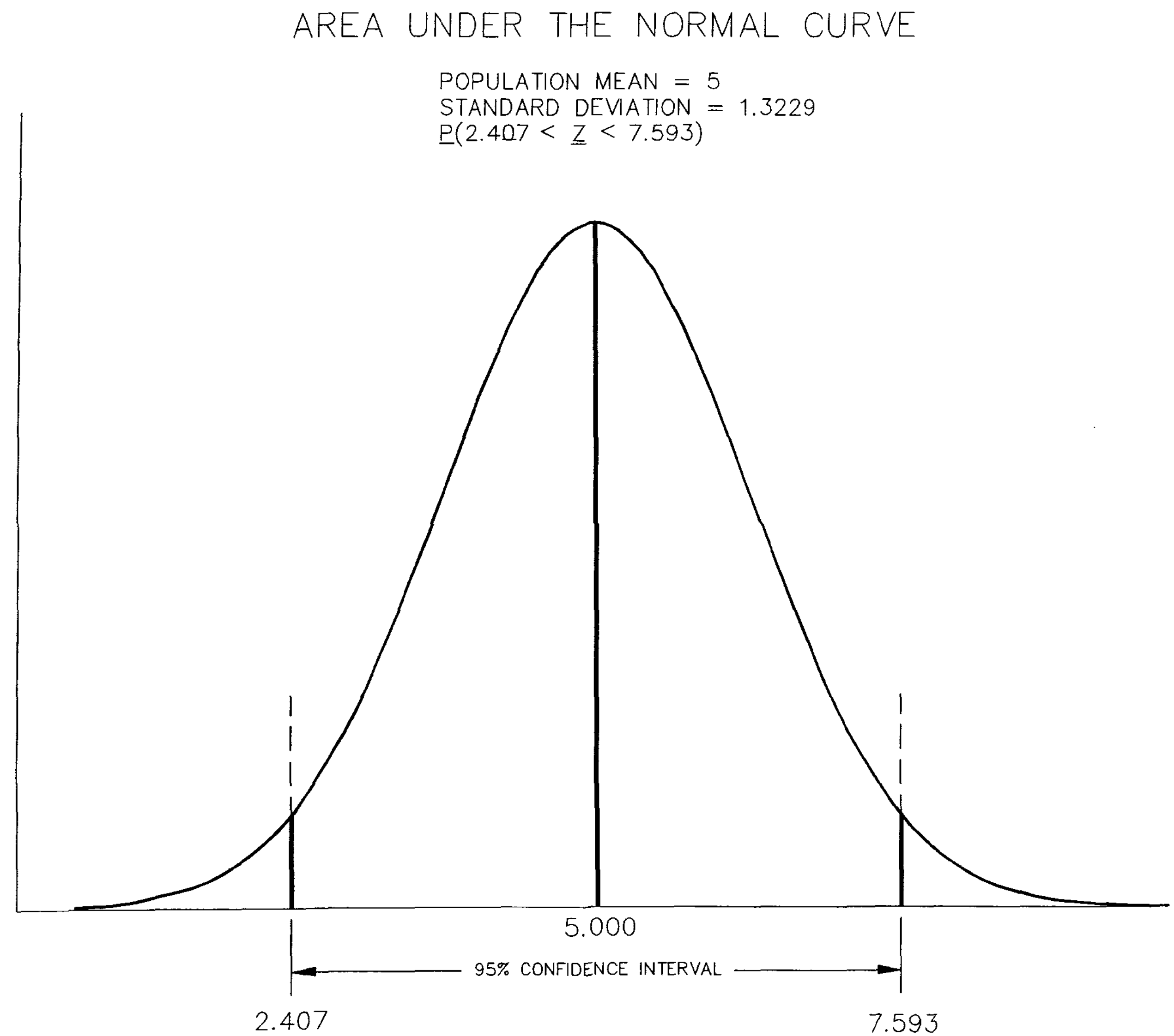

AREA UNDER THE NORMAL CURVE
POPULATION MEAN = 5
STANDARD DEVIATION = 1.3229
P(2.407 < Z < 7.593)
5.000
95% CONFIDENCE INTERVAL
2.407
7.593

FIGURE 4.3

Western Illinois University in teaching process control. The sampling box has a population comprised of the following :

population size (N) = 1,000 bearings

yellow plated bearings = 150/1,000 = .15 = 15%

black plated bearings = 100/1,00 = .10 or 10%

nonplated bearings = 750/1,000 = .75 or 75%.

The population proportion (p) for plated bearings is given as:

p = 250/1,000 = .25 or 25%.

Because population characteristics such as the proportion (p) of plated bearings are known, they are considered population parameters. These are contrasted with sample statistics which are estimates of population parameters.

BINOMIAL DISTRIBUTION

The binomial distribution is one of several discrete probability distributions which find widespread use in industrial settings. Two assumptions must first be made before satisfying the requirements of the binomial distribution.

The first assumption is that the probability of the occurrence of an event and that the nonoccurrence of the event remain constant from one trial to the next (independence). This can be satisfied by requiring that the population be at least ten times greater than the sample size. The second assumption requires independence in that the occurrence of an event does not preclude the occurrence of a subsequent event.

Knowing that p equals the probability of occurrence and that 1-p equals the probability of nonoccurrence, we can determine the probability of x occurrences in n trials by using the following formula .

$$P(x \text{ occurs in } n \text{ trials}) = C_x^n \, p^x \, (1-p)^{n-x} \tag{4.7}$$

where :
p = the probability of occurence
$1-p$ = the probability of nonoccurence
n = the sample size
x = the number of occurence in n trials
$C_x^n = \dfrac{n!}{x!(n-x)!}$

The expected value for the random variable X in n successive trials is:

$$E(X) = n\,p \tag{4.8}$$

The variance of the distribution is:

$$Var(X) = n\,p\,(1-p) \tag{4.9}$$

It follows that the standard deviation is:

$$\sigma_{\underline{x}} = \sqrt{\underline{n}\ \underline{p}\ (1 - \underline{p})} \tag{4.10}$$

Example. Suppose that a process is known to operate at a fraction defective of .25 or 25%. If a sample of twenty-five pieces is taken randomly from the population, what is the probability that:

1) no pieces are defective

2) one piece is defective

3) more than one piece is defective?

Using the sampling box, one can simulate the process if we use Equation 4.7 and let;

$\underline{P}(0)$ = no colored bearings

$\underline{P}(1)$ = one colored bearing

$\underline{P}(\underline{x}) \rangle 1$ = more-than-one bearing.

Where: $\underline{n}$ = 25 and $\underline{p}$ = .25.

$$\underline{P}(0) = C_0^{25}\underline{p}^0(1-.25)^{25-0}$$
$$= \frac{25!}{(0)!(25-0)!}(1)(.75)^{25} = .0002 = .02\%$$

$$\underline{P}(1) = C_1^{25}\underline{p}^1(1-.25)^{25-1}$$
$$= \frac{25!}{(1)!(25-1)!}(.25)^1(.75)^{24} = .0063 = .63\%$$

Because we know that the sum of all probabilities must be equal to one, we can subtract the probabilities of zero and one to yield the probability that there is greater than one colored bearing in the sample. For example:

1- (.0002 + .0063) = .9935. Thus, the probability that x is greater than one is 99.4%.

Figure 4.4 shows the frequency distribution associated with the last example. The highlighted probabilities on the left side of Figure 4.4 represent the probability of obtaining zero and one colored bearings from the sampling box for this sample size (n = 25). The portion of the distribution which is not highlighted represents the probabilities of obtaining other than zero or one colored bearings; the sum of which is 99.4%.

From Equation 4.9, the variance of this distribution is:

$$\mathrm{var}(X) = (6.25)(1\text{-}25) = 4.6875$$

From Equation 4.10, the standard deviation is:

$$\sqrt{(6.25)(1-25)} = 2.1651+$$

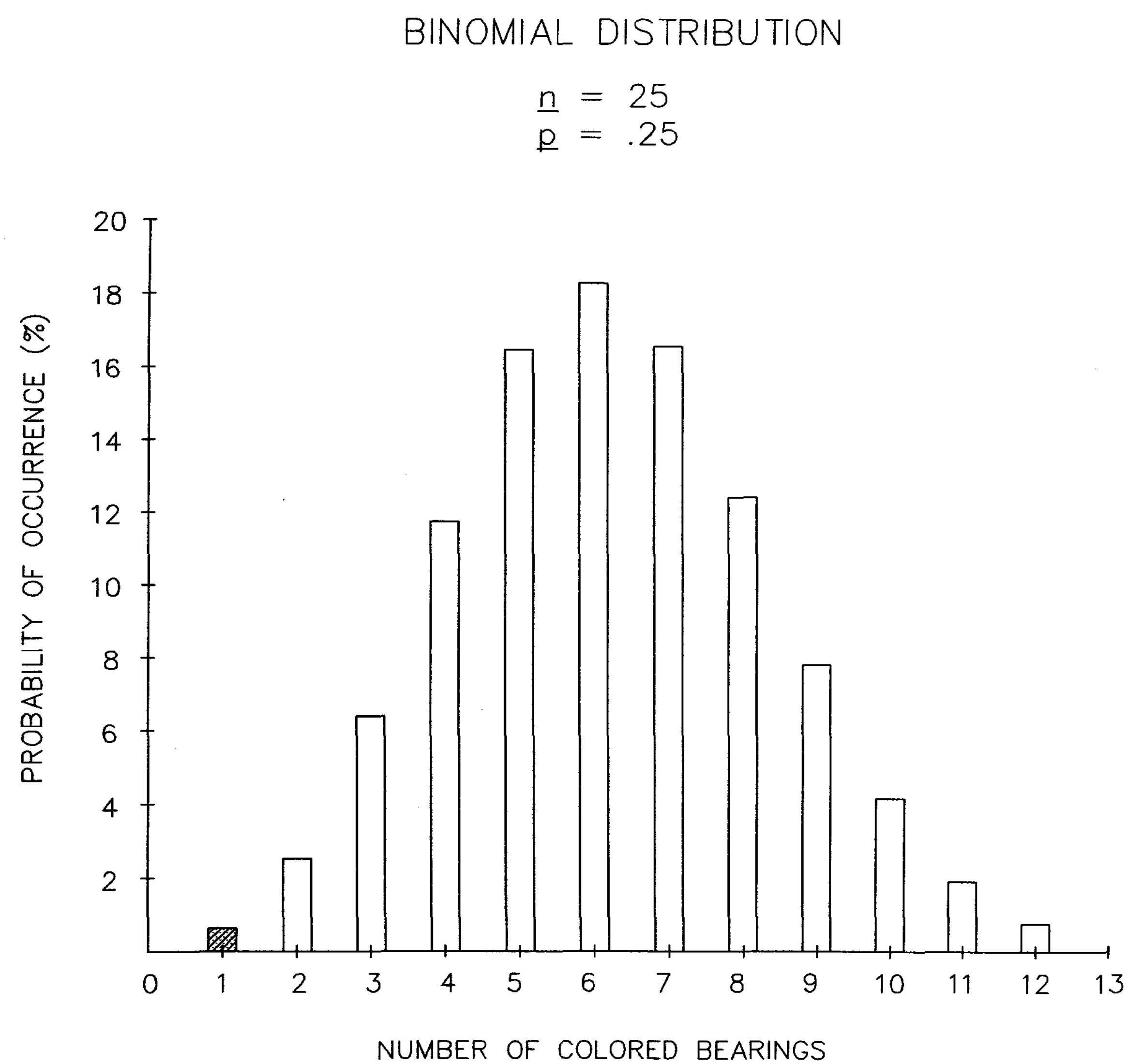

BINOMIAL DISTRIBUTION
n = 25
p = .25
PROBABILITY OF OCCURRENCE (%)
20
18
16
14
12
10
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
NUMBER OF COLORED BEARINGS

FIGURE 4.4

CONFIDENCE INTERVAL FOR PROPORTIONS IN A BINOMIAL DISTRIBUTION

If the sample size is relatively large with ($n \geq 25$) with $n(p)$ and $n(1-p)$ both greater than or equal to 5, the following formula may be utilized as a very good normal approximation to the binomial random variable:

$$p \pm Z_{\alpha/2} \sqrt{\frac{p\,(1-p)}{n}} \qquad (4.11)$$

where: p = proportion of occurrences
n = sample size
$Z_{\alpha/2}$ = standard normaldistribution value (2-sided limits)

Please note that the following expression used above is also known as the standard error estimate for the sample size n in the sampling distribution:

$$\sqrt{\frac{p\,(1-p)}{n}}$$

Example. Consider a process which is known to be binomially distributed with a population proportion (p) equal to .10. What is the ninety-five percent confidence interval for the true population proportion?

Because both (n)p and n(1-p) are greater than or equal to five and sample size (n) is greater than or equal to twenty five, we can use Equation 4.11 as an approximation.

Where: n = 50 ; α = .05

p = .10 ; $\alpha/2$ = .025 (two-tailed limits)

$Z_{\alpha/2}$ = 1.96 (Appendix B-1)

Substituting these values into Equation 4.11 yields the following:

$$.10 \pm 1.96 \frac{\sqrt{(.10)(1-.10)}}{50} = .10 \pm .083$$

After calculation, the confidence interval becomes: upper confidence limit for the proportion (UCLp) = .183; lower confidence limit for the proportion (LCLp) = .017.

Using this formula, in the long run, we can expect that ninety-five percent of the intervals will contain the true population proportion and five percent will not. Figure 4.5 shows the ninety-five percent confidence interval for this sampling distribution as the probabilities bounded by the two vertical lines.

95% CONFIDENCE INTERVAL
FOR
BINOMIAL DISTRIBUTION

$\underline{n} = 50$
$\underline{\bar{p}} = .10$

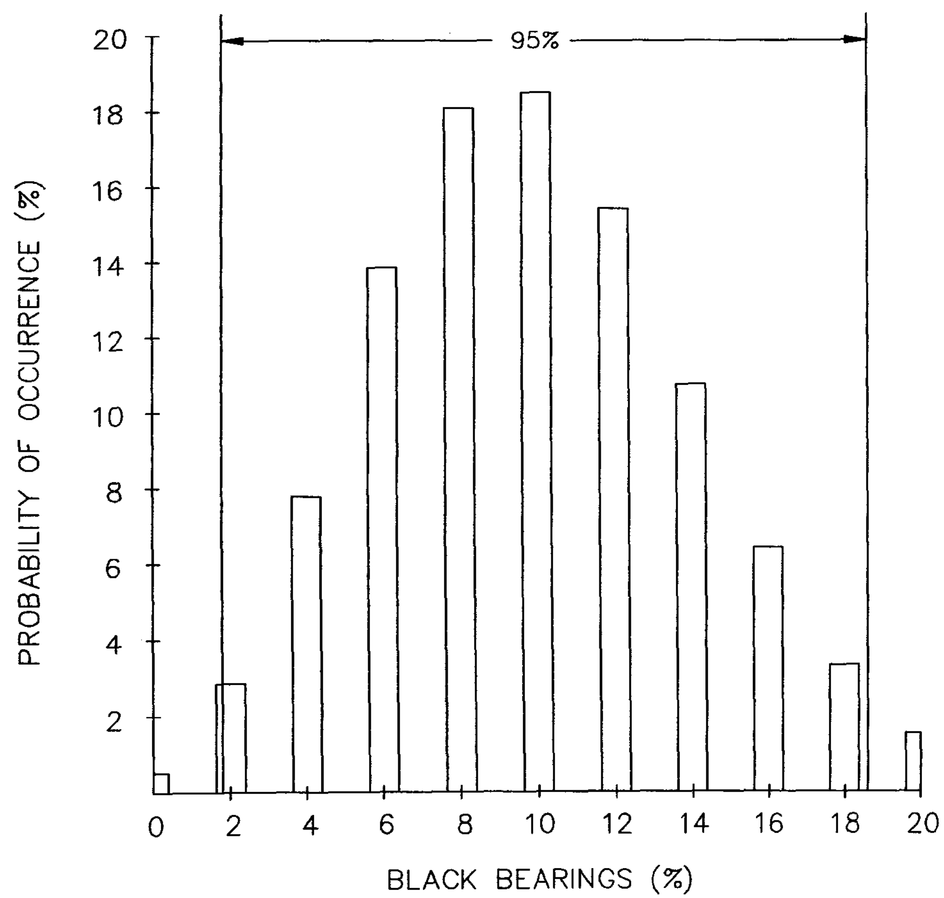


FIGURE 4.5

If the sample size is small and the conditions prescribed in the previous example are not met [$n \leq 25$ and/or either $n\,p$ or $n(1-p)$ are ⟨5], the following confidence interval formulas can be used.

The lower limit LCLp is calculated:

$$LCLp = \frac{2r-1-Z^2_{\alpha/2}-Z_{\alpha/2}\sqrt{[\frac{(2r-1)(2n-2r+1)}{n}]+Z^2_{\alpha/2}}}{2(n+Z^2_{\alpha/2})} \qquad (4.12)$$

The upper limit UCLp is calculated:

$$UCLp = \frac{2r+1+Z^2_{\alpha/2}+Z_{\alpha/2}\sqrt{[\frac{(2r+1)(2n-2r-1)}{n}]+Z^2_{\alpha/2}}}{2(n+Z^2_{\alpha/2})} \qquad (4.13)$$

where: r = expected number of occurances in the sample
n = sample size
$Z_{\alpha/2}$ = two-tailed confidence limit value for the normal distribution (Appendix B-1)

Example. If, in the last example, the sample size was only twenty-five pieces, then $n\,p$ = (.10)(25) or 2.5 would not satisfy the requirement of Equation 4.22. We can, however, use Equation 4.12 to establish the ninety-five percent confidence interval for the population proportion ($\underline{p}$) where:

$\underline{r}$ = 3 (expected number of occurrences in the sample)

$\underline{n}$ = 25

$\underline{Z}_{\alpha/2}$ = 1.96 (Appendix B-1)

When these values are substituted into Equation 4.12, the lower limit (LCL$\underline{p}$) becomes:

$$\underline{p} = \frac{6-1-(1.96)^2-1.96\sqrt{[\frac{(6-1)(50-6+1)}{25}]+(1.96)^2}}{2[25+(1.96)^2]} = -.07$$

Because there can never be less then zero occurrences in any sample taken, we then set the lower control limit LCL$\underline{p}$ = 0.

We can calculate the upper control limit for this distribution using Equation 4.13 as follows:

$$UCL\underline{p} = \frac{6+1+(1.96)^2+1.96\sqrt{[\frac{(6+1)(50-6-1)}{25}]+(1.96)^2}}{2[25+(1.96)^2]} = .32$$

The upper control limit for the population proportion ($\underline{p}$) is thirty-two percent.

HYPERGEOMETRIC PROBABILITY

If the discrete random variable comes from a finite population of size N and sampling without replacement takes place, then the hypergeometric probability mass function can give more exact predictions about population characteristics (parameters).

The probability of obtaining x occurrences in a sample taken (without replacement) from a finite population can be determined by the use of the following formula:

$$P(x) = \frac{C_{n-x}^{N-M} \, C_{x}^{M}}{C_{n}^{N}} \tag{4.14}$$

where: M = number of possible occurances in the population
N = population size
n = number of items (or trials) taken without replacement
x = number of occurances in the n trials

If the hypergeometric random variable (X) comes from a population of N items containing two groups which are mutually exclusive and collectively exhaustive, then:

the expected value $E(x) = \frac{(n)(M)}{N}$ (4.15) and the variance is :

$$\underline{V}(\underline{X}) = \frac{\underline{N}-\underline{n}}{\underline{N}-1}\left[\underline{n}\left(\frac{\underline{M}}{\underline{N}}\right)\left(1-\frac{\underline{M}}{\underline{N}}\right)\right] \tag{4.16}$$

Example. Suppose that a sample of twenty-five pieces is taken from a population of finite size (N = 100). Using Equation 4.14, determine the probability of obtaining zero, one and greater-than-one defects.

Where: N = 100; M = 10; n = 25, the number of defects obtained during sampling are represented by black bearings.

Substituting the above values into Equation 4.14, the expression becomes (for the following conditions):

$$\underline{P}(0) = \frac{C_{25-0}^{100-10}\; C_{0}^{10}}{C_{25}^{100}}$$

$$= \frac{\dfrac{(90)!}{(25)!(65)!}\;\dfrac{(10)!}{(0)!(10-0)!}}{\dfrac{(100)!}{(25)!(100-25)!}}$$

$$= .0479 \quad or \quad 4.79\%$$

$$\underline{P}(1) = \frac{C_{25-1}^{100-10} \; C_{1}^{10}}{C_{25}^{100}}$$

$$= \frac{\dfrac{(90)!}{(24)!(90-24)!} \; \dfrac{(10)!}{(1)!(10-1)!}}{\dfrac{(100)!}{(25)!(100-25)!}}$$

$$= .1813 \quad or \quad 18.13\%$$

$\underline{P}(\underline{x}$ greater than one) = 1 - (.0479+.1813)

= .7708 or 77.08%.

Figure 4.6 shows the frequency distribution for this example. The highlighted probabilities represent the probability of obtaining zero and one black bearings for this sampling distribution. The portion of this distribution which is not highlighted represents the probabilities associated with obtaining other than zero or one occurrence; the sum of these probabilities is 77.08%.

From Equation 4.15, the expected value for this hypergeometric random variable is:

$$\underline{E}(\underline{x}) = \frac{(25)(10)}{100} = 2.5$$

HYPERGEOMETRIC DISTRIBUTION

$\underline{N} = 100$

$\underline{n} = 25$

$\overline{\underline{p}} = .10$

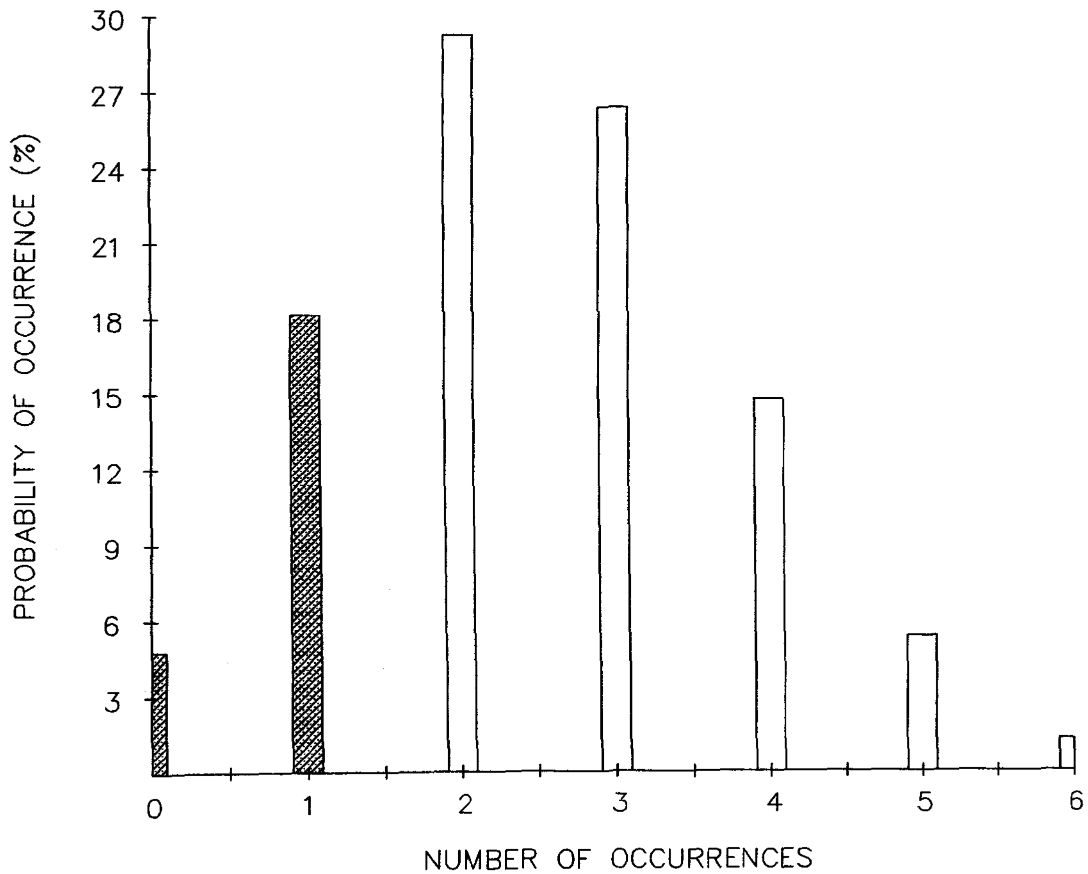


FIGURE 4.6

From Equation 4.16, the variance is:

$$\underline{V}(\underline{X}) = \frac{100-25}{100-1}[25(\frac{10}{100})(1-\frac{10}{100})] = 1.70.$$

Although the population size is finite (1,000 pcs.), it is very large for hypergeometric formula calculations. The previous example manipulated the population parameters associated with the Sampling Box to yield the values used. In this case, the number of black bearings (found) less than five were considered as being zero and the number of bearings (found) greater than five but less than fifteen were considered as one, etc...This type of scenario can be used in a classroom setting for demonstration purposes however, the population size mandates rather unwieldy calculations.

CONFIDENCE INTERVAL DETERMINATION AS A NORMAL APPROXIMATION TO THE HYPERGEOMETRIC DISTRIBUTION

When the universe is manageably small, the hypergeometric distribution can be used to determine confidence bounds. However, when both the sample and universe are large in size, the hypergeometric distribution becomes very burdensome to calculate. As the binomial distribution was approximated by the normal distribution, we can do much the same to approximate the hypergeometric distribution as shown below:

$$\underline{p} \pm \underline{Z}_{\alpha/2} \sqrt{\frac{\underline{p}\,(1-\underline{p})}{\underline{n}}\,(1-\frac{\underline{n}}{\underline{N}})} \qquad (4.17)$$

where: $\underline{N}$ = universe size
$\underline{Z}_{\alpha/2}$ = standard normal value for a given confidence level specified by α with the risk divided between the two tails (Appendix B-1)

Example. Using the same data that was presented in the last section, calculate the ninety-five percent confidence limits for this hypergeometric distribution based on Equation 4.17.

Where: $\underline{N} = 100$; $\alpha = .05$

$\underline{n} = 25$; $\alpha/2 = .025$

$\underline{p} = .10$; $\underline{Z}_{\alpha/2} = 1.96$ (Appendix B.1)

Substituting these values into Equation 4.17 we get:

$$.10 \pm 1.96 \sqrt{\frac{.10\,(1-.10)}{25}\,(1-\frac{25}{100})}\ .$$

After calculation, these limits become:

LCLp = .20

LCLp = .00

The results calculated above indicate that if a twenty-five piece sample is taken from this hypergeometric distribution, with a known probability of occurence on any one trial set at ten percent, then this normal approximation shows the ninety-five percent probability of including the expected number of occurrences.

CHI-SQUARE DISTRIBUTION

The chi-square distribution is of particular interest to process control engineers for confidence interval determinations and hypothesis testing. This distribution is widely used for drawing inferences about populations because of its relative ease of use and the volume of information obtainable through knowledge gained during experimentation.

The chi-square random variable (χ^2) is a continuous probability distribution which is unimodal and is skewed toward the right tail of the distribution. The amount of skewness the chi-square random variable exhibits depends upon the degrees of freedom (df or ν) (see Appendix C) in the distribution.

A random variable is considered as being chi-square if its probability density function (pdf) is as presented below:

$$Y = \frac{e^{\frac{-\chi^2}{2}} (\chi^2)^{\frac{(\nu-2)}{2}}}{2^{\frac{\nu}{2}} (\frac{\nu-2}{2})!} \tag{4.18}$$

where: χ^2 = the chi-square random variable
ν = df = degrees of freedom
e = a numeric constant (2.718+) commonly found in mathemathics
y= the ordinate of the function

Example. The ordinate for the chi-square random variable (0.5) for four degrees of freedom, determined by using Equation 4.18, is given as:

$$Y = \frac{e^{\frac{-0.5^2}{2}} (0.5^2)^{\frac{(4-2)}{2}}}{2^{\frac{4}{2}} (\frac{4-2}{2})!} = .1103$$

This value is the circled point in Figure 4.7. The reader will notice that this chi-square distribution is sharply skewed to the right.

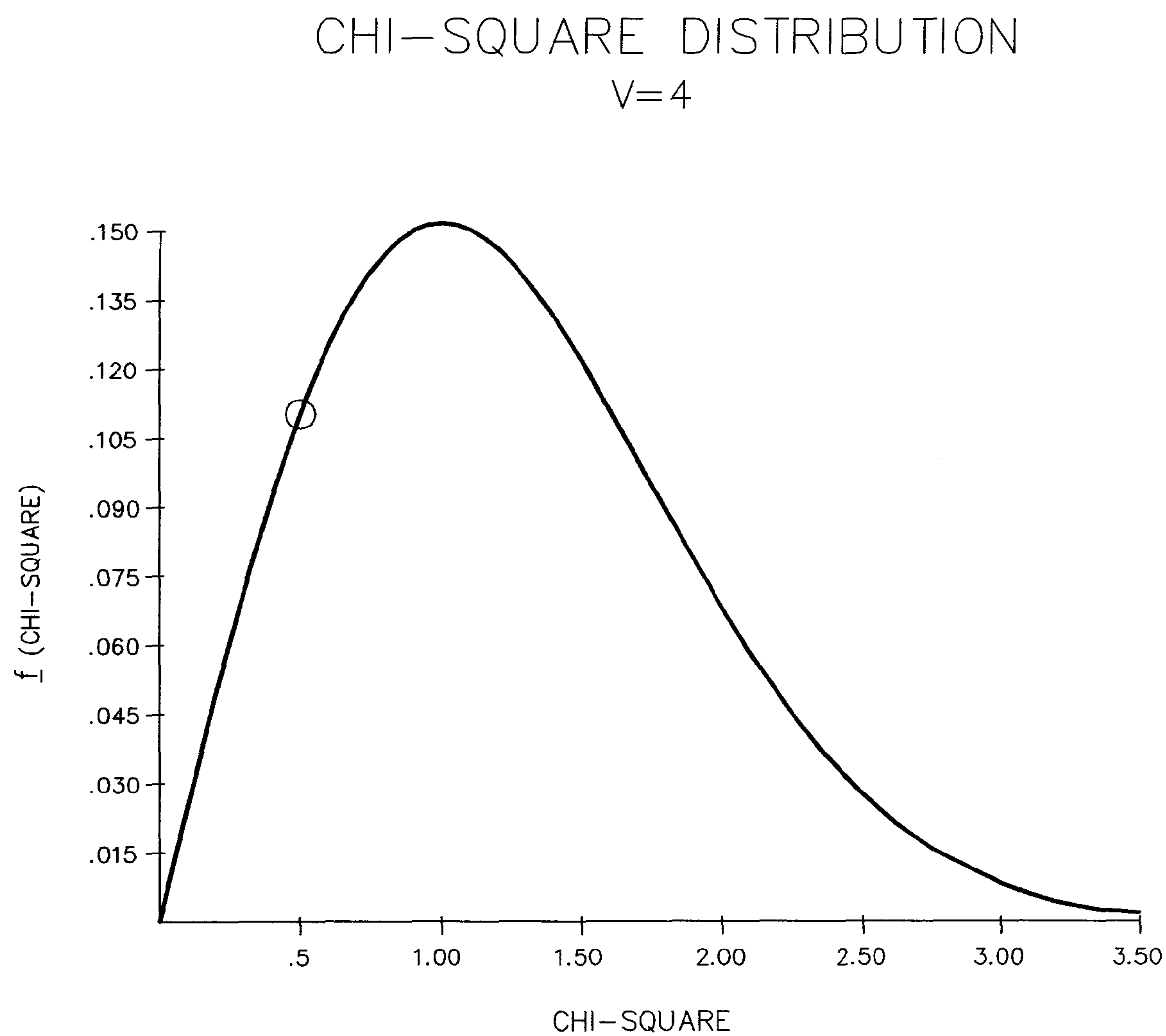

CHI-SQUARE DISTRIBUTION
V=4
.150
.135
.120
.105
.090
.075
.060
.045
.030
.015
f (CHI-SQUARE)
.5
1.00
1.50
2.00
2.50
3.00
3.50
CHI-SQUARE

FIGURE 4.7

The standardized random variable of a chi-square distribution is given as:

$$\chi^2 = \frac{(\underline{n}-1)\ \underline{s}^2}{\sigma^2} \tag{4.19}$$

where: $\underline{n}$ = sample size
$\underline{s}^2$ = sample variance
σ^2 = population variance

The sampling distribution of the chi-square random variable for an interval estimate at a confidence level (1-α) divided between both tails is:

$$\frac{(\underline{n}-1)\underline{s}^2}{\chi^2_{\underline{U};\underline{n}-1}} \leq \sigma^2 \leq \frac{(\underline{n}-1)\underline{s}^2}{\chi^2_{\underline{L};\underline{n}-1}} \tag{4.20}$$

where: $\chi^2_{\underline{L};\underline{n}-1}$ and $\chi^2_{\underline{U};\underline{n}-1}$ are the lower and upper chi-square values respectively; each with (n-1) degrees of freedom (from Appendix B-2)

Example. Suppose that the sampling box is known to contain twenty-five percent colored bearings. If the box produces samples which are truly random, then the ninety-five percent confidence interval for a twenty-five piece sample would be as is shown below:

Where: $\underline{p} = 0.25$

$\underline{s}^2 = (25)(0.25)(0.75) = 4.6875$

$\underline{n} = 25$

$\alpha/2 = 0.025$

$\chi^2_{\underline{U};24} = 12.4$ (Appendix B-2)

$\chi^2_{L;24} = 39.4$ (Appendix B-2).

After substitution into Equation 4.20, this expression becomes:

$$\frac{(24)(4.6875)}{12.4} \preceq \sigma^2 \preceq \frac{(24)(4.6875)}{39.4}$$

$$or\ 9.07 \preceq \sigma^2 \preceq 2.86$$

This result tells the experimenter that the ninety-five percent confidence interval for the variance of a twenty-five piece sample, taken from the population, is contained within these two values. Note that the sample variance (4.6875) does fall between these two values, as it is used in the estimate.

POISSON DISTRIBUTION

The Poisson distribution plays an important role in discrete probability statistics and is particularly useful in acceptance sampling. Several conditions must be met before $\underline{X}$ can be considered a Poisson random variable.

1. The number of occurrences are independent and remain constant from trial to trial.
2. The probability of the occurrence of an event is very small relative to the total of all possible like occurrences.

The conditions for the Poisson distribution are met when small values of ($\underline{X}$) exist [ex., $\underline{P}(\underline{X}) \leq .10$] and when sample size ($\underline{n}$) is large (ex., $\underline{n} \geq 16$). A population size ($\underline{N}$) which is at least ten times greater than the sample size further helps to satisfy Poisson distribution requirements.

The Poisson probability mass function for the random variable $\underline{X}$ is:

$$\underline{P}(\underline{x}) = \frac{e^{-\mu}\mu^{-\underline{x}}}{\underline{x}!} \qquad (4.21)$$

where: e is the constant (2.718+) which is the limit of $(1+\frac{1}{\underline{X}})^{\underline{X}}$ as $\underline{X}$ approach infinity.

The expected number of occurrences is given by:

$$\underline{E}(\underline{x}) = \mu \tag{4.22}$$

The variance is denoted by:

$$Var\ (\underline{x}) = \mu \tag{4.23}$$

The expected number of occurrences in a sample is taken to be $\underline{x}$.

Example. If a process is known to operate at a ten percent defect rate, what is the probability of obtaining zero, one and greater-than-one defects; given that the sample size is fifty pieces?

Let: $\underline{n} = 50$

$\underline{p} = .10$

$\mu = \underline{n}\ \underline{p} = (.10)(50)$ or 5

Note that the number of black bearings selected represents the number of defects.

Substituting these values into Equation 4.21, we get the following:

$$\underline{P}(0) = \frac{e^5 5^0}{0!} = .0067\ or\ .67\%$$

$$\underline{P}(1) = \frac{e^5 5^1}{1!} = .0337\ or\ 3.37\%$$

$\underline{P}(\underline{x}$ greater than one) = 1 - (.0067+.0337) = .9596 or 95.96%.

The Poisson distribution for this example is shown in Figure 4.8. The highlighted probabilities in this sampling distribution are those

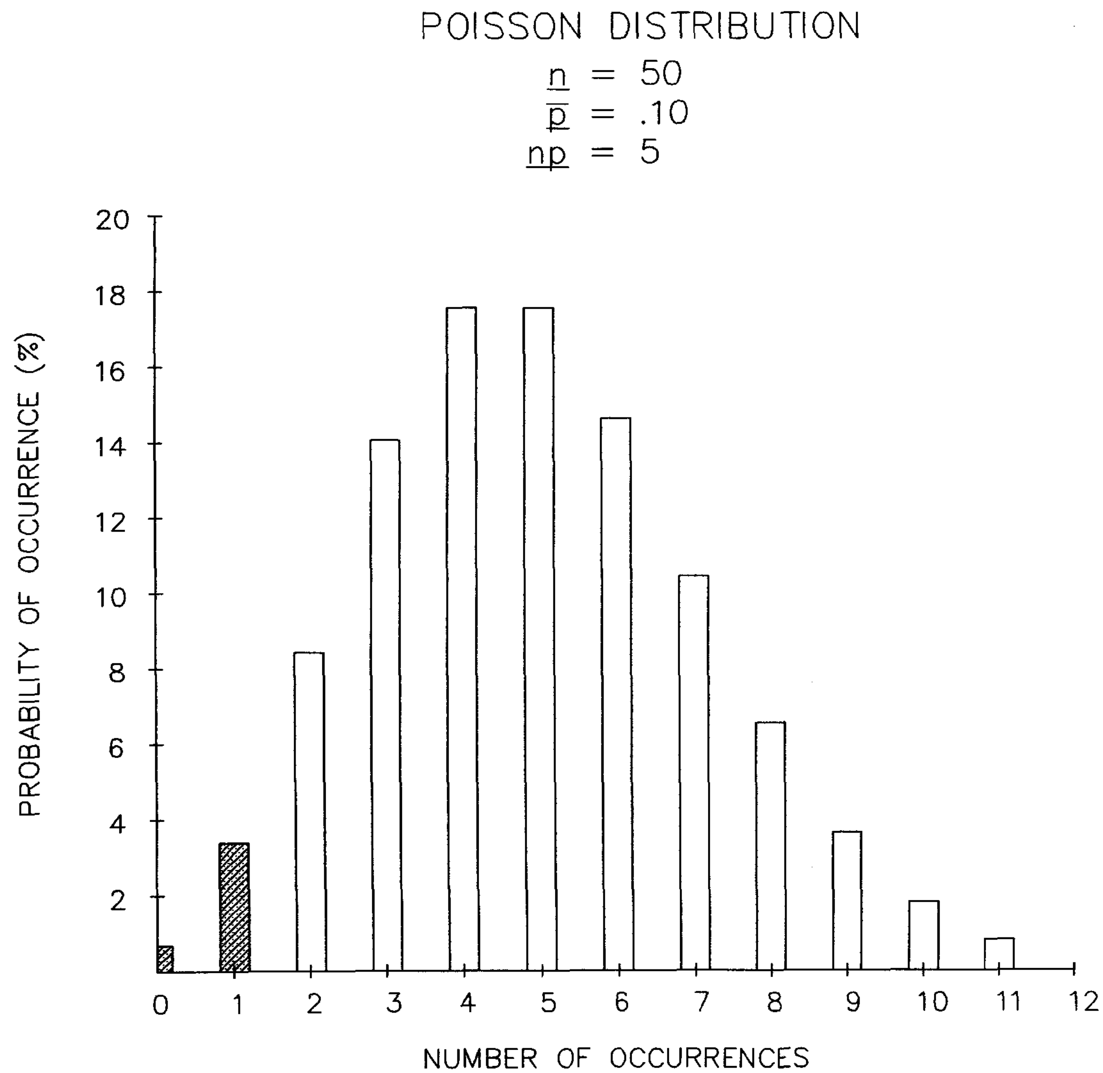

POISSON DISTRIBUTION
n = 50
p̄ = .10
np = 5
PROBABILITY OF OCCURRENCE (%)
20
18
16
14
12
10
8
6
4
2
0 1 2 3 4 5 6 7 8 9 10 11 12
NUMBER OF OCCURRENCES

FIGURE 4.8

representing the probability of occurrence for zero and one black bearings obtained from the sampling box for this sample size (n = 50). The probabilities of obtaining greater than one black bearing are shown as the non-highlighted probabilities. The probability of greater-than-one black bearing can be viewed as the sum of the non-highlighted probabilities and is equal to 95.96%.

From Equation 4.22, we know that the mean (μ) is $\underline{n}\ \underline{p} = 5$ and that from Equation 4.23 the variance of $\underline{X}$ is also 5.

DETERMINATION OF CONFIDENCE INTERVALS FOR THE POISSON RANDOM VARIABLE

Calculation of Poisson distribution data confidence intervals can be readily performed by using a chi-square table approximation method supported by R. A. Dovich. "To calculate the upper confidence limit for the number of occurrences, calculate the appropriate degrees of freedom (df) for the chi-square table as $\underline{\nu} = 2(\underline{r}+1)$. To calculate the lower confidence limit for number of occurrences, calculate the $\underline{\nu}$ as $2\underline{r}$."[2]

---

[2]Robert A. Dovich, Quality Engineering Statistics (Milwaukee, WI: ASQC Press, 1992), p.14.

From above, the upper confidence limit is determined by using Appendix B-2 and the following:

$$\underline{\nu} = 2(\underline{r}+1) \tag{4.24}$$

The lower confidence limit is generated by the use of Appendix B-2 and the following equation:

$$\underline{\nu} = 2\underline{r} \tag{4.25}$$

Example. In an earlier section, it was determined that a process was operating at a ten percent defect rate. If a sample of fifty pieces is to be taken, what is the ninety-six percent confidence interval for the number of expected occurrences in the sample using Equation 4.24 and 4.25?

Let: $\underline{n} = 50$

$\underline{p} = .10$

$\underline{r} = \underline{n}\ \underline{p}$ or 5

and the number of black bearings selected represents the number of defects.

The degrees of freedom ($\underline{\nu}$) for the upper confidence limit are taken as:

$\underline{\nu} = 2(5+1)$ or 12.

Because we are concerned with determining a ninety-six percent confidence limit divided between two tails, the chi-square table value (see Table 4.4) for alpha of .02; df or $\underline{v}$ = 12 is 24.054. Dividing this value by two, our upper confidence limit becomes 12.03 occurrences. The lower confidence limit for the sampling distribution of this process is calculated from :

$\underline{v}$ = 2(5) or 10.

The chi-square table value (see Table 4.4) for $\underline{v}$ = 10 and .98 or ($1-\alpha/2$) is 3.059. Dividing this by two we get 1.53 occurrences.

Using the sampling box in a classroom setting, a student would not normally be expected to obtain more than twelve or less than one black bearing in a fifty piece sample ninety-six percent of the time.

TESTS OF HYPOTHESIS

All of the concepts presented to this point are oriented toward empowering the practicioner with the tools necessary to quantify process quality and provide a basis for decision making. These tools are necessary for drawing inferences about point estimates of the mean or probabilities associated within specific values within a distribution and the nature of process variability, as it relates to the dispersion of data from a given population.

Distribution of Chi-Square

df	0.99	0.98	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	0.0157	0.0628	0.00393	0.0158	0.0642	0.148	0.455	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	0.0201	0.0404	0.103	0.211	0.446	0.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210	13.815
3	0.115	0.185	0.352	0.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.341	16.268
4	0.297	0.429	0.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277	18.465
5	0.554	0.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086	20.517
6	0.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812	22.457
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475	24.322
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090	26.125
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666	27.877
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209	29.588
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725	31.264
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217	32.909
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688	34.528
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141	36.123
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578	37.697
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.663	32.000	39.252
17	6.408	7.255	8.762	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409	40.790
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	25.869	32.346	34.805	42.312
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191	43.820
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566	45.315
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932	46.797
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289	48.268
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638	49.728
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980	51.179
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314	52.620
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642	54.052
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963	55.476
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278	56.893
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588	58.302
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892	59.703

Table 4.4

Another means of drawing inferences about populations is through the use of hypothesis testing. This approach involves conceptualizing a process by taking samples from a population and comparing them to a hypothesized value or performing comparisons of different populations to determine the degree of compatibility or similarity between them. Hypothesis testing has widespread applicability where experimentation is concerned and is particularly useful for decision making in nearly all manufacturing settings.

The following sections highlight some of the most commonly used forms of hypothesis testing and does so with the assumption that the reader has at least some prior knowledge of hypothesis testing and how it relates to process control. This is done in part to focus on how the sampling box may be used in classroom settings. Moreover, the following sections provide some of the examples this researcher has found most applicable in using the sampling box for teaching manufacturing process control.

Tests of hypotheses generally start with a hypothesized population value; which may be based on past or historical data. A value obtained during experimentation, by taking samples from the population, is then compared to the hypothesized value to determine if a difference exists between two population values.

The hypothesized value is the basis for comparison and hence, becomes the standard for the null hypothesis (H_0). When a test of hypothesis is performed and there is no discernable difference between the two population values (lack of significance), the experimenter fails to reject the null hypothesis. If the assumed or known population value, obtained through experimentation, differs noticeably from the hypothesized value (significance exists); then the experimenter rejects the null hypotheses (H_0) in favor of the alternate hypotheses (H_A). Tests of hypotheses take one of three different forms which are presented below.

Form I : H_0: $p_1 = p_2$

H_A: $p_1 \neq p_2$

This is a two-tailed test to determine if the two values are not significantly different (could be equal). The decision to reject H_0 (Accept H_A) is made if the test statistic (Z) falls outside the critical region given by $Z_{\alpha/2}$.

Form II : H_0: $p_1 \geq p_2$

H_A: $p_1 < p_2$

This is a one-tailed test to determine if population one ($\underline{p}_1$) is greater than or equal to population two ($\underline{p}_2$) or $\underline{H}_0$ is not rejected. The decision to reject the null hypothesis (Accept $\underline{H}_A$) is made if $\underline{p}_1$ is less than the critical value ($\underline{Z}_\alpha$).

Form III : $\underline{H}_0$: $\underline{p}_1 \leq \underline{p}_2$

$\underline{H}_A$: $\underline{p}_1 \rangle \underline{p}_2$

This is a one-tail test to determine if population one ($\underline{p}_1$) is less than or equal to population two ($\underline{p}_2$) or $\underline{H}_0$ is rejected. The decision to reject the null hypothesis (Accept $\underline{H}_A$) is made if $\underline{p}_1$ is greater than $\underline{p}_2$ at the critical value ($\underline{Z}_\alpha$).

With any test of hypothesis, there is always the risk that an incorrect decision has been made. Assuming that the samples are taken in a random manner (devoid of sampling bias) the associated risks are presented in Table 4.5.

Note that in Table 4.5 failure to reject the null hypothesis when it is true carries a high probability (1-α). This is desired to help assure that the correct decision has been made. This is known as a strong

DECISION RISKS FOR HYPOTHESIS TESTING

Decision	Actual State of nature (reality)	
	$\underline{H}_0$ is true	$\underline{H}_\Delta$ is true
$\underline{H}_0$ Do Not Reject	Correct decision $\underline{p} = (1-\alpha)$	Incorrect decision $\underline{p} = \beta$ Type II error
$\underline{H}_\Delta$ Reject	Incorrect decision Type I error (α)	Correct decision Correct decision $\underline{p} = (1-\beta)$

Table 4.5

decision because it requires (by nature) that reasonable proof of significance be presented before rejection of $\underline{H}_0$.

Rejecting the null hypothesis (Accepting $\underline{H}_A$) when $\underline{H}_0$ is true results in a type one error and is represented by alpha (α). The value of alpha is generally set at a small probability, in part, so that the probability of committing this type of error maintains a small risk. A graphical depiction of these relationships is shown in Figure 4.9. The area of the distribution, bounded by the action limits, is determined by the significance level. In this particular case, the confidence level $(1-\alpha)$ is ninety-five percent and represents the strong decision. Because this curve represents a two-tailed test, the probability of committing a type one error (5%) is divided between the two tails ($\alpha/2 = 2.5\%$) and is shown as the rejection region at both extremities of the distribution.

Failure to reject the null hypothesis ($\underline{H}_0$) when the alternate hypothesis ($\underline{H}_A$) is true (ie. $\underline{H}_0$ is false), results in an incorrect decision or a type two error denoted by beta (β). This is known as the weak decision. The reason for this is because one cannot wholly declare that the true population value is indeed equal to the hypothesized value. This point makes sense when one considers the fact that the true population

ACTION LIMITS FOR TWO-TAILED TEST OF HYPOTHESES: ALPHA = .05

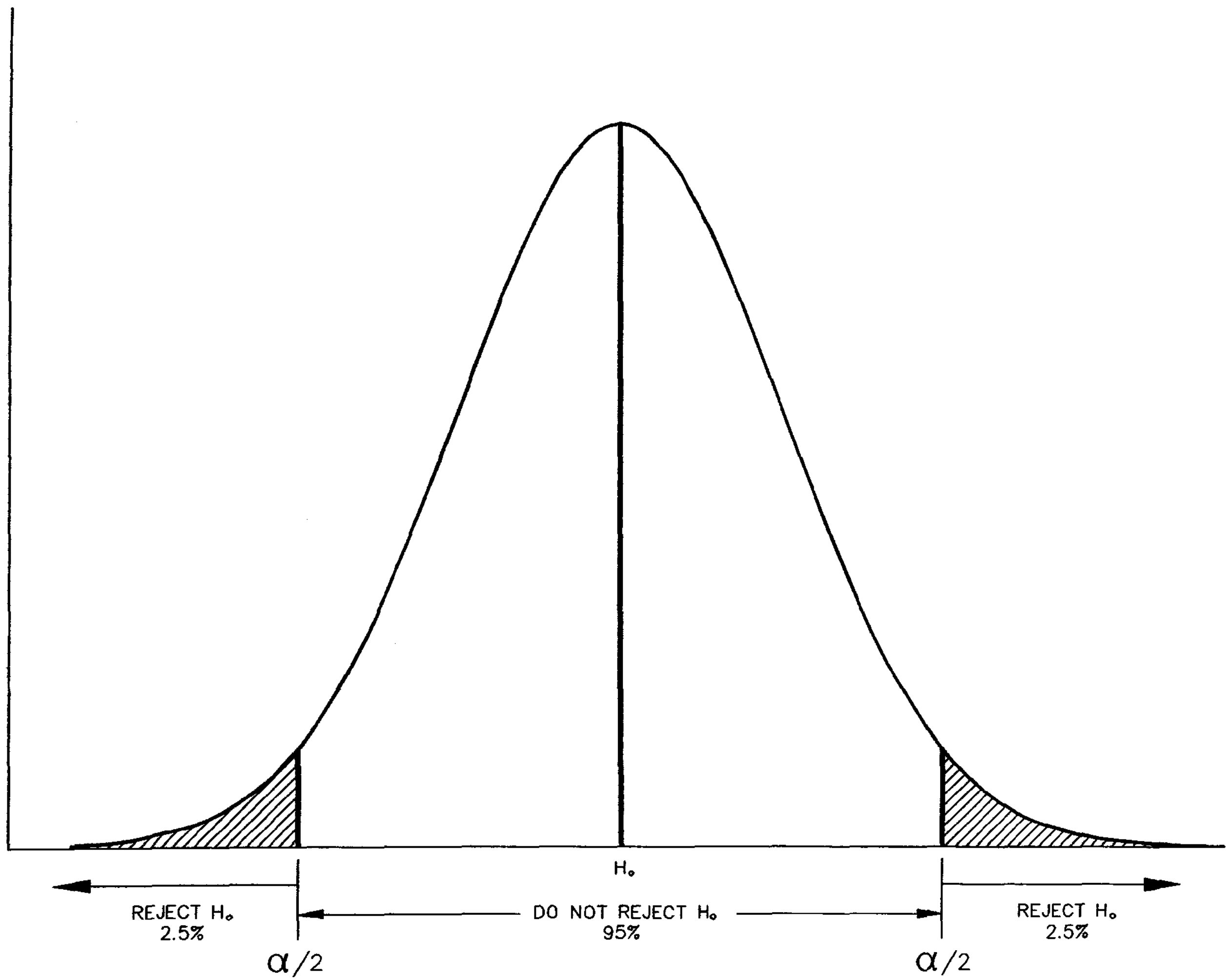


FIGURE 4.9

value may be contained within the action limits but not be exactly equal to the hypothesized value.

Rejecting the null hypothesis ($\underline{H}_0$) when the alternate hypothesis is true results in a correct decision and is denoted by 1-β. Recognizing a difference when one actually exists is also known as the power of the test.

An example of how the type two error (β) may appear in a test of hypothesis is shown in Figure 4.10. This may be viewed by first recognizing the action limits associated with the null hypothesis (μ_0). Please notice that part of the distribution associated with the assumed value (μ_Δ) or true population mean (if it is actually true) falls within these action limits. The area of this curve contained within the acceptance region is the probability of committing a type two error and is, therefore, the area of the beta (β) risk. This area is highlighted for the convenience of the reader. The regions not highlighted, at both extremities of the same curve, represent the power of the test (1-β). More will be presented on the power of a test in a later section.

Once the decision to conduct a test of hypothesis has been made, the experimenter must state the null hypothesis and the alternate hypothesis using one of the three forms previously shown. The next step

BETA RISK FOR TWO-TAILED TEST OF HYPOTHESIS: ALPHA = .05

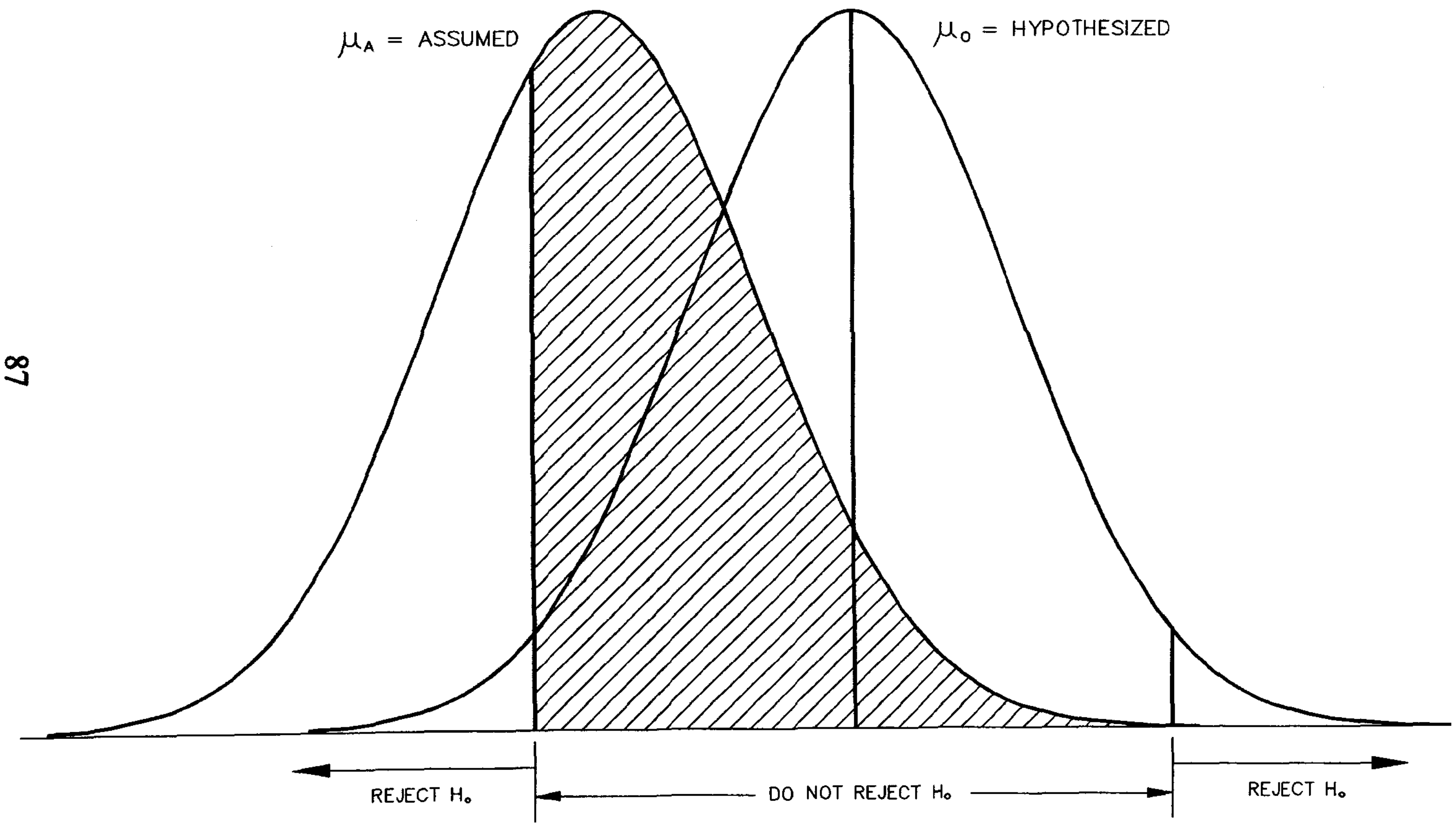


FIGURE 4.10

is to select the test statistic upon which a decision rule is based. In the examples that follow, the population proportion (p) is used as the test statistic.

Step three involves setting the value of alpha for the type one risk. This step is necessary because, theoretically, a distribution may continue through infinity and some point must practicably be selected as a point where significance exists. From this, a decision rule can be established at a given confidence level, determined by alpha, along with the action limits associated with the test. For the cases that follow, the significance level is arbitrarily set at five percent ($\alpha = .05$). The decision to set alpha at a specific value is up to the experimenter to decide, based on the risk considered optimal for the circumstances.

After completion of the previous steps, the experimenter can next take a sample of size n from the population, compute the sample test statistic (p_x) and compare it to the action limits. The decision to reject H_0 or not to reject H_0, based on the action limits, can then be made.

There are several types of hypothesis tests for common use when testing for differences in population proportions or count data. Each test is briefly presented then followed with an example; with population values relating directly to those associated with the Sampling Box.

The tests of hypotheses illustrated in this text are intended to show significance. This is done in order to prove that the population proportions associated with the Sampling Box can be tested for a detectable difference. This may be important to the instructor who wishes for students to detect a population difference and hence, significance. In the classroom, the students will unwittingly prove what the instructor already knows; that significance exists between population values.

This "instructor knowledge" can be used to standardize the specific tests the students are to perform. The population values were selected with the premise that this condition exist; not only for the previously mentioned reasons but also for aiding in the simplification of grading by the instructor. For example, if the instructor knows the population parameters and that significance is expected, it becomes easier to grade the work of a student. This point will hold true when testing for differences between black bearings (10%) against colored bearings (25%).

HYPOTHESIS TESTING COMPARING A SAMPLE PROPORTION p_o (HYPOTHESIZED) TO A KNOWN OR ASSUMED POPULATION PROPORTION p

Comparison of sample data to a population proportion can be performed utilizing the standardized test statistic presented below if the population random variable is presumed as being normally distributed.

Standard normal test statistic:

$$Z = \frac{p - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}} \qquad (4.26)$$

This is based on the condition that both $n\ p$ and $n(1-p)$ are greater than or equal to 5.

Example. Suppose that a student is told that the Sampling Box contains ten percent colored bearings (p_o) and is instructed to test if the population of colored bearings is equal to this hypothesized value at the ninety-five percent confidence level. The student is also asked to test this hypothesis by taking a one-hundred piece sample from the population at a given p and using Equation 4.26. The student should yield results somewhat analogous to those presented below:

Form I test	Action Limits

$H_0 : p = p_o$

$H_A : p \neq p_o$ $\qquad \pm 1.96$

$\alpha \ = .05$

$\alpha/2 = .025$(two-tailed test) Decision rule

$Z_{\alpha/2} = 1.96$(Appendix B-1) Reject H_0 if $Z > |1.96|$

Do not reject H_0 if $Z \leq |1.96|$

$p_o = .10$

$p \ = .25$

$n \ = 100.$

Using Equation 4.26 to calculate Z yields:

$$Z = \frac{.25 - .10}{\sqrt{\frac{.10(1-.10)}{100}}} = 5.00$$

Because the test statistic ($Z = 5.00$) falls well beyond $Z = 1.96$, the student rejects the null hypothesis (H_0), accepts the alternate hypothesis (H_A)and concludes that a significant difference exists at the ninety-five percent confidence level.

Example. Suppose that, in the last example, the student was instructed to test whether or not the assumed or true population proportion (p) was greater than or equal to the hypothesized value (p_0). Using the same population values and Equation 4.26, the student may have been expected to yield results similar to those presented below.

Form II test	Action Limit
H_0: $p \geq p_0$	
H_A: $p < p_0$	-1.645 standard units
α = .05 (one-tailed test)	Decision rule
Z_α = 1.645 (Appendix B-1)	Reject H_0 if $Z < -1.645$.
	Do not reject H_0 if $Z \geq -1.645$.

p_0 = .10

p = .25

n = 100

Using Equation 4.26 to calculate Z yields:

$$Z = \frac{.25 - .10}{\sqrt{\frac{.10(1-.10)}{100}}} = 5.00$$

Notice that this was a one-tail test with all of the risk (α) associated with the left tail. Because the concern was only that the population proportion (p) not be less than the hypothesized value, the student failed to reject the null hypothesis (H_0) at the ninety-five percent confidence level.

HYPOTHESIS TESTING FOR DIFFERENCES BETWEEN TWO POPULATION PROPORTIONS

Comparing two population proportions for differences when the sample sizes are large[i.e. both $n\ p$ and $n(1 - p)$ are ≥ 5] can be performed utilizing the standard normal distribution and the following group of equations:

$$Z = \frac{p_1 - p_2}{S_{p_1 - p_2}} \tag{4.27}$$

where:

$$S_{p_1 - p_2} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \tag{4.28}$$

$$\hat{p} = \frac{(n_1)(p_1) + (n_2)(p_2)}{n_1 + n_2} \tag{4.29}$$

Example. Consider an experiment where an instructor asks two students to obtain samples from the Sampling Box for the sake of comparison.

One student is assigned the task of determining the proportion of colored bearings (p_1) taken from a seventy-five piece sample and the other student is responsible for determining the proportion of black bearings (p_2) from a one-hundred piece sample taken from the population. The students are then instructed to compare the two population values to determine if a significant difference exists between the respective proportions (assuming equality). Using Equation 4.27, 4.28 and 4.29, while testing at the ninety-five percent confidence level, the student's test should result similarly to that shown below.

Form I test	Action Limits
H_0: $p = p_o$	
H_A: $p \neq p_o$	$\pm$ 1.96
α = .05	
$\alpha/2$ = .025(two-tailed test)	Decision rule
$Z_{\alpha/2}$ = 1.96(Appendix B-1)	Reject H_0 if $Z \geq \lvert 1.96 \rvert$
	Do not reject H_0 if $Z \leq \lvert 1.96 \rvert$

Student One	Student Two
$n_1 = 75$	$n_2 = 100$
colored = 19	black = 10
p_1 = 19/75 or .25	p_2 = 10/100 or .10

Note: $\underline{p}_1$ is rounded for simplicity in illustration. Substituting these values into Equation 4.29 yields:

$$\hat{\underline{p}} = \frac{(75)(.25)+(100)(.10)}{75+100} = .164 \ or .16$$

Upon insertion into Equation 4.28, leaves:

$$\underline{S}_{\underline{p}_1 - \underline{p}_2} = \sqrt{(.16)(1-.16)(\frac{1}{75}+\frac{1}{100})} = .056 \ or \ .06$$

Equation 4.27 now becomes:

$$\underline{Z} = \frac{-25 \ - \ .10}{.06} = 2.5$$

Since the calculated test statistic ($\underline{Z}$ = 2.5) is greater than $\underline{Z}$ = 1.96, the null hypothesis ($\underline{H}_0$) is rejected, the alternate hypothesis ($\underline{H}_A$) is accepted and the students declare that a difference exists at the ninety-five percent confidence level.

HYPOTHESIS TESTING FOR DIFFERENCES BETWEEN COUNT DATA

If the number of occurrences of an event, per unit of sample, is of primary concern (rather than proportion), tests for differences between count data can be performed. If the number of units inspected or the number of occurrences are large (as defined in section 3.1), the standard normal distribution can be used for comparing test statistics.

When the sample sizes for count data comparison are equal, the following formula can be used as a two-tailed test for equality of count data.

$$\underline{Z} = \frac{\left| \underline{Y}_1 - \underline{Y}_2 \right| - .5}{\sqrt{\underline{Y}_1 + \underline{Y}_2}} \tag{4.30}$$

where: $\underline{Y}_1$ = number of occurances in sample one
$\underline{Y}_2$ = number of occurances in sample two
.5 = continuity correction factor (see Appendix C; Definitions of Terms)

When unequal sample sizes are used, the previous formula is modified for use as follows:

$$\underline{Z} = \frac{\underline{n}_2 \underline{Y}_1 - \underline{n}_1 \underline{Y}_2}{\sqrt{(\underline{n}_1 \underline{n}_2)}\sqrt{\underline{Y}_1 + \underline{Y}_2}} \tag{4.31}$$

Example. Utilizing the same population characteristics presented in the last section, an instructor assigns one student the responsibility for determining the number of colored bearings ($\underline{Y}_1$) taken from the population contained within the Sampling Box. The second student is instructed to count the number of black bearings ($\underline{Y}_2$) obtained from the Sampling Box. Both students are given the charge of taking one-hundred piece samples and then directed to test whether population one ($\underline{Y}_1$) is greater than or equal to population two ($\underline{Y}_2$) at the ninety-five percent confidence level. Using Equation 4.30, a likely result; similar to that which follows, should be shown.

Form II test	Action Limit
$\underline{H}_0 : \underline{p} \geq \underline{p}_0$	
$\underline{H}_A : \underline{p} < \underline{p}_0$	-1.645 standard units
α = .05 (one-tailed test)	Decision rule
$\underline{Z}_\alpha$ = 1.645 (Appendix B-1)	Reject $\underline{H}_0$ if $\underline{Z} < -1.645$.
	Do not reject $\underline{H}_0$ if $\underline{Z} \geq -1.645$.

Student one	Student two
$\underline{Y}_1 = 25$	$\underline{Y}_2 = 10$

Upon substitution into Equation 4.30, gives:

$$\underline{Z} = \frac{|25-10|-.5}{\sqrt{25+10}} = 2.45$$

Because the test statistic ($Z = 2.45$) is not less than $Z = -1.645$, the decision is made not to reject the null hypothesis; concluding that significance in the test did not exist and that population one (Y_1) is greater than or equal to population two (Y_2) at the ninety-five percent confidence level.

Example. If student one in the last section was instructed to take a seventy-five piece sample from the population of colored bearings (Y_1) and student two was instructed to take a one-hundred piece sample from the population of black bearings (Y_2), using Equation 4.31, the test results may have been as follows:

Form II test	Action Limit
$H_0 : p \geq p_0$	
$H_A : p < p_0$	-1.645 standard units
α = .05 (one-tailed test)	Decision rule
Z_α = 1.645 (Appendix B-1)	Reject H_0 if $Z < -1.645$.
	Do not reject H_0 if $Z \geq -1.645$.

Student one	Student two
$n_1 = 75$	$n_2 = 100$
$Y_1 = 19$	$Y_2 = 10$.

After substitution into Equation 4.31, the expression becomes:

$$\underline{Z} = \frac{(100)(19)-(75)(10)}{\sqrt{(75)(100)}\ \sqrt{19+10}} = 2.47$$

Since the value of the test statistic ($\underline{Z}$ = 2.47) is not less than $\underline{Z}$ = -1.645, the null hypothesis ($\underline{H}_0$) is not rejected. From this, it is concluded that population one ($\underline{Y}_1$) is greater than or equal to population two ($\underline{Y}_2$) at the ninety-five percent confidence level.

SAMPLE SIZE SELECTION FOR TESTS OF HYPOTHESIS

As proper sample size selection is imperative for estimating the population proportion for acceptance sampling and control charting purposes; it is equally important when performing tests of hypothesis. In the preceding sections on hypothesis testing, sample sizes were selected arbitrarily; primarily because the focus of these sections was on conducting these tests and how they may be applied in a classroom setting. However, in an industrial setting, this practice may prove unsatisfactory. It is for this reason that sample size selection for hypothesis testing is now presented.

To this point, selection of the alpha (α) risk and the confidence level was of primary concern however, by specifying a tolerable level for the beta (β) risk as well, the maximum risk probabilities can be set to increase the credibility of a test. For example, setting the value of alpha at five percent assures the practicioner that there is a ninety-five percent probability that the null hypothesis will not be rejected when it is true.

By setting the beta (β) risk at, for example, .10; the experimenter has established a ten percent risk that the null hypothesis ($\underline{H}_0$) will be rejected (accept $\underline{H}_A$) when it is, in fact, true. Moreover, this also sets the probability that the null hypothesis ($\underline{H}_0$) will be rejected when it is, in fact, false ($\underline{H}_A$ is true). In this case, it represents a ninety percent chance that the correct decision has been made. This is referred to as the power of the test (1-β) and will be treated in greater detail later.

It becomes increasingly clear why the proper selection of sample sizes for hypothesis testing is important to the practicioner. By balancing the risks associated with a test against the cost of an incorrect decision, one can be reasonably confident that the maximum benefit and credibility can be gained from a test of hypothesis.

Several examples of sample size selection models are presented, in the sections that follow, with examples of how they may apply to teaching the merits of the Sampling Box.

SAMPLE SIZE SELECTION FOR TESTING $H_0: P_2 = P_1$ AGAINST $H_A: P_2 \neq P_1$

The sample size requirement for this test is given as:

$$n = \frac{\left[Z_{\alpha/2} \sqrt{p_2(1-p_2)} + Z_{\beta} \sqrt{p_1(1-p_1)} \right]^2}{d^2} \qquad (4.32)$$

where: p_2 = the hypothesized value of p and
p_1 is the assumed true value of p
$\alpha/2 = p\,(z \geq Z_{\alpha/2})$
$\beta = p\,(z \geq Z_{\beta})$
$d = (p_1 - p_2)$; a difference detected with a given power $(1-\beta)$

Example. Consider some of the previously used population parameters which were set forth relating to the sampling box. If it is desired to test the proportion of colored bearings (p_1) against the proportion of black bearings (p_2) at a ninety-five percent confidence level with the power of the test (1-β) set at ninety percent, the experimenter can use Equation 4.32 as follows:

$p_1 = .25$

$p_2 = .10$ $\qquad$ $\beta = .10$

$\alpha = .05$ $\qquad$ $Z_{\beta} = 1.282$ (Appendix B-1)

$\alpha/2$ = .025 (two-tailed test) $\underline{d}$ = .15

$\underline{Z}_{\alpha/2}$ = 1.96 (Appendix B-1)

Incorporating the necessary substitutions into Equation 4.32 leaves:

$$\underline{n} = \frac{[1.96\sqrt{(.10)(.90)} + (1.282)\sqrt{(.25)(.75)}]^2}{(.15)^2} = 58.1 \text{ or } 59$$

Note : always round up to the next larger integer.

Thus, the minimum number of pieces required for this test is fifty-nine. The reader will notice that this requirement has been surpassed in all previous tests.

SAMPLE SIZE SELECTION FOR TESTING $\underline{H}_0$: $\underline{P}_1 \geq \underline{P}_2$ OR $\underline{H}_0$: $\underline{P}_1 \leq \underline{P}_2$

The sample size requirement for this test is given as:

$$\underline{n} = \frac{[\,\underline{Z}_\alpha\sqrt{\underline{p}_2(1-\underline{p}_2)} + \underline{Z}_\beta\sqrt{\underline{p}_1(1-\underline{p}_1)}\,]^2}{\underline{d}^2} \tag{4.33}$$

where: $\underline{p}_1$ is the hypothesized value of $\underline{p}$ and
$\underline{p}_2$ is the assumed true value of $\underline{p}$
$\alpha = \underline{p}(\underline{z} \geq \underline{Z}_\alpha)$
$\beta = \underline{p}(\underline{z} \geq \underline{Z}_\beta)$
$\underline{d} = (\underline{p}_2 - \underline{p}_1)$; a difference detected with a given power $(1-\beta)$

Example. Again; using the same conditions set forth in the last example and setting $\alpha = .05$ and $\beta = .10$, determine the minimum sample size required for testing the proportion of colored bearings ($\underline{p}_1 = .25$) against the proportion of black bearings ($\underline{p}_2 = .10$). The solution is as presented below:

$\underline{p}_1 = .25$ $\beta = .10$

$\underline{p}_2 = .10$ $\underline{Z}_\beta = 1.282$ (Appendix B-1)

$\alpha = .05$ $\underline{d} = .15$

$\underline{Z}_\alpha = 1.645$ (Appendix B-1)

Making the necessary substitutions into Equation 4.33 results in:

$$\underline{n} = \frac{[\ (1.645)\sqrt{(.10)(.90)} + (1.282)\sqrt{(.25)(.75)}\ ]^2}{(.15)^2}$$

$$= 48.9 \textit{ or } 49$$

The minimum number of pieces required for this test is forty-nine.

If the instructor using this box wishes to test for differences between black bearings (10%) against yellow bearings (15%), as the population is now configured, setting alpha (α) at five percent and beta (β) at ten percent would require a sample size of 438 bearings (when testing for equality). This may appear to present a problem for the sampling box; which is designed to provide a maximum of a one-hundred piece sample size. However, only the hypergeometric distribution makes the assumption that sampling without replacement takes place. This can ultimately infer that four samples of one-hundred bearings can be taken, followed by a fifty-piece sample; of which only the first thirty-eight are counted from the last sample. This sampling practice is acceptable and is recommended by this researcher.

Another approach toward testing for differences between black and yellow bearings could be to set the population proportion of black bearings at five percent, if a test of equality is desired. If all of the following conditions are set (i.e. $\alpha = .05$ and $\beta = .10$; $\underline{H}_0 = .05$), then the requirement would be seventy-nine bearings tested. This is well within the capabilities of the sampling box and may provide an alternative testing scenario for students to perform.

CHI-SQUARE GOODNESS-OF-FIT TEST

One of the most important uses of the chi-square distribution is to help determine whether a random variable is of a specific theorized distribution. This is particularly true in testing for normality in the distribution of data in a population. This use of the chi-square distribution is called the goodness-of-fit test and the formula for generating the test statistic is given by:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \; ; \; (i = 1,2,3,...k). \qquad (4.34)$$

where:

O_i = observed frequency in class number i
E_i = expected frequency for class number i
k = the number of classes

This chi-square test has (K-m-1) degrees of freedom. If the population parameter(s) is/are not known, then m represents the number which must be estimated.

The basis of the chi-square goodness-of-fit test is taking the difference between the observed and the expected frequencies for each respective class, squaring them, then dividing them by their expected values. The sum of these squared deviations are then compared to a standard tabular value to determine if the data represent (or fit) the hypothesized probability distribution.

Intuitively, one will recognize that if the data represent a perfect fit, the chi-square test statistic value will be zero. Conversely, the chi-square distribution has an upper bound which represents a complete lack of fit with the hypothesized distribution. For this reason, the chi-square goodness-of-fit test has an upper-tail rejection region determined largely by the level of significance (α) of the test.

Example. It is hypothesized that the expected frequency or number of colored bearings in each of four classes is 6.25 (see Figure 4.11). The instructor wishes for the students to test whether each class of samples produces random results which are equal to this hypothesized value ($\underline{H}_O$: frequency = 6.25) at the ninety-five percent confidence level. The alternate hypothesis being that the four cells do not operate in the prescribed random manner ($\underline{H}_A$: frequency $\neq$ 6.25). Table 4.6 shows the results of this test using Equation 4.34.

where:

α = .05 (one-tailed test)

$\underline{k}$ = 4

$\underline{m}$ = 0

$\underline{df}$ = 4-0-1 = 3

$\chi^2_{.05;0.3}$ = 7.82 (Appendix B-2)

Because the calculated value (1.08) of the chi-square statistic is less than the tabular value (7.82), the students fail to reject the null hypothesis and conclude that the data fit well with the hypothesized value (6.25). Were each of these classes considered parallel manufacturing processes, it could have been stated that there was no

CLASSES FOR CHI-SQUARE GOODNESS-OF-FIT TEST

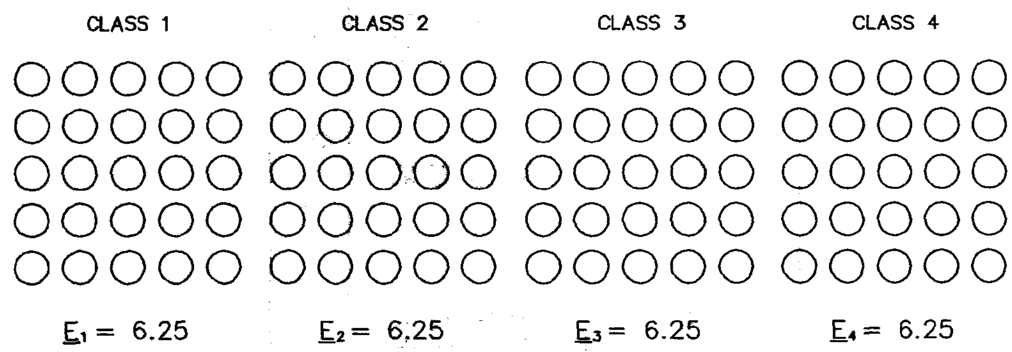


FIGURE 4.11

CALCULATION OF CHI-SQUARE FOR GOODNESS-OF-FIT TEST

Class i		O_i	E_i	(O_i-E_i)	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
I	colored	5	6.25	-1.25	1.5625	0.25
II	colored	8	6.25	1.75	3.0625	0.49
III	colored	5	6.25	-1.25	1.5625	0.25
IV	colored	7	6.25	0.75	0.5625	0.09
Totals		25	25	0		$\chi^2 = 1.08$

Table 4.6

detectable proof that any one process performed differently than another.

The form of this particular chi-square distribution is very similar to that shown in Figure 4.7; differing only by one degree of freedom (3 df vs 4 df).

CHI-SQUARE GOODNESS-OF-FIT SAMPLE SIZE REQUIREMENTS

Because the chi-square goodness-of-fit test has only an upper limit rejection boundary, the alternative hypothesis values are either greater-than or less-than those tested against as indicated below:

1. $\chi^2_O \succ \chi^2_H$ or $\sigma^2_O \succ \sigma^2_H$
2. $\chi^2_O \prec \chi^2_H$ or $\sigma^2_O \prec \sigma^2_H$

where : χ^2_H = the hypothesized variance
χ^2_O = the variance observed during testing

Dovich specifies two cases where these values are applicable. In case one, a predetermined difference in variances is specified; detectable at a given beta (β) risk. A ratio (R) is then taken of the standard deviations (rather than variances). This ratio is taken as follows:

$$\underline{R} = \frac{\sigma_{\underline{o}}}{\sigma_{\underline{H}}} \tag{4.35}$$

A sample size for a given $\underline{R}$ value, α and β is determined by:

$$\underline{n} = 1 + \frac{1}{2}\left[\frac{Z_{1-\alpha} + \underline{R}(Z_{1-\beta})}{\underline{R}-1}\right]^2 \tag{4.36}$$

Example. It has been hypothesized that the variance (σ_H^2) associated with the proportion of black bearings contained within the Sampling Box is 2.25. However, one student in the class contends that the true variance of this population is 3.19 (σ_O^2) and is therefore, larger than the hypothesized value. The instructor decides to arbitrate the case and instructs the students to perform a chi-square test to prove whether or not this new population variance (3.19) is indeed greater than the hypothesized population variance (2.25). After considerable debate, the students decide to set the type one risk (α = .10) and the type two risk (β = .15) for the test. What is the minimum number of sample pieces required to perform this test using Equation 4.35 and 4.36?

From above, it is given that:

$\underline{H}_O : 2.25 \geq 3.19$

$\underline{H}_A : 2.25 \langle\ 3.19$

α = .10; $\underline{Z}_{\alpha}$ = 1.282(Appendix B-1)

β = .15; $\underline{Z}_{\beta}$ = 1.037(Appendix B-1)

Case one test (one-tail) with the alternate hypothesis 2.25 ⟨ 3.19.

$$\sigma_0^2 = \sqrt{3.19} = 1.786$$
$$\sigma_{\underline{H}}^2 = \sqrt{2.25} = 1.500$$

Substituting these values into Equation 4.35 to determine $\underline{R}$ yields:

$$\underline{R} = \frac{1.786}{1.500} = 1.191$$

Calculation of the sample size using Equation 4.36 then becomes:

$$\underline{n} = 1 + \frac{1}{2}\left[\frac{(1.282) + (1.191)(1.037)}{(1.191-1)}\right]^2$$
$$= 87.8 \ or \ 88$$

The minimum sample size required to perform this test is eighty-eight pieces.

Case two exists when the observed variance is presumed to be less than the hypothesized value. This presumption leads to a calculation of $\underline{R}$ which is less than one. The primary difference in the two cases can be viewed in the denominator where the probability changes to (1-$\underline{R}$) as shown below:

$$\underline{n} = 1 + \frac{1}{2}\left[\frac{\underline{Z}_{1-\alpha} + \underline{R}(\underline{Z}_{1-\beta})}{1-\underline{R}}\right]^2 \tag{4.37}$$

Example. Suppose that the variance of the sampling distribution of black bearings within the Sampling Box is hypothesized to be 6.75. The assertion however, is that the true variance of the sampling distribution is actually less than or equal to the hypothesized value and is given as 4.5. Again, the instructor directs the students to perform a chi-square test to determine if this new population variance (4.5) is less than the hypothesized value (6.75). Using the same type one error (α = .10) risk and the same type two error risk (β = .15), as was presented in the last example, the students determine the minimum sample size by way of Equation 4.35 and 4.37 and is represented below.

From above, it is given that:

$\underline{H}_0$: 6.75 ≥ 4.5

$\underline{H}_A$: 6.75 〈 4.5

α = .10; $\underline{Z}_\alpha$ = 1.282 (Appendix B-1)

β = .15; $\underline{Z}_\beta$ = 1.037 (Appendix B-1)

Case two test (one-tail) with the alternate hypothesis 6.75 〉 4.5.

$$\sigma_{\underline{0}}^2 = \sqrt{4.5} = 2.121$$

$$\sigma_{\underline{H}}^2 = \sqrt{6.75} = 2.598$$

Substituting these values into Equation 4.35 to determine $\underline{R}$ gives:

$$\underline{R} = \frac{(2.121)}{(2.598)} = 0.816$$

Equation 4.37 then becomes:

$$\underline{n} = 1 + \frac{1}{2}\left[\frac{(1.282) + (0.816)(1.037)}{(1-0.816)}\right]^2 = 67.9 \text{ or } 68 \text{ pcs.}$$

The minimum size requirement for this test is sixty-eight pieces.

POWER FUNCTION

In the last section, the concept of the power of the test (1-β) was introduced. This represents the probability that the null hypothesis ($\underline{H}_0$) will be rejected when it is, in fact, false. It is also the complement of beta (β); the incorrect decision to accept the null hypothesis when it is false. If it is desired to determine the power of a test, it must be recognized that the true population mean ($\underline{p}$) falls within the do-not-reject $\underline{H}_0$ region and it is almost never certain exactly where the true population is within this range.

One can, in a hypothetical sense, calculate the power of a test if one considers an assumed true population mean value. This can be performed for a variety of assumed values within the population to reveal the likelihood, at a given value, what the power of the test is. Recognizing this, the power of a test can be determined by the following formula:

$$\text{Power} = \underline{P}\,[\underline{Z} < \frac{(LTL)-\underline{p}}{\sigma} \text{ or } \underline{Z} > \frac{(UTL)-\underline{p}}{\sigma}] \qquad (4.38)$$

where: $\underline{p}$ = the assumed true population mean
σ = the standard deviation of the population mean
LTL = lower test limit: $[\underline{p}_0 - (\underline{Z}_{\alpha/2})(\sigma)]$
UTL = upper test limit: $[\underline{p}_0 + (\underline{Z}_{\alpha/2})(\sigma)]$
$\underline{Z}$ = the standard normal value for the test calculated above
$\underline{P}$ = the sum of the probabilities for the values beyond $\underline{Z}$.

Example. It has been hypothesized that a one-thousand piece population contains 250 colored bearings. If a sample of one-hundred bearings is pulled from the population, with the confidence level of the test set at ninety-five percent (α = .05), what is the power of the test if the true proportion is actually 110 colored bearings? Assume that sigma (σ) equals 4.3 and use Equation 4.38.

$\underline{p}_0 = 25$ LTL = 25 - (1.96)(4.3) = 16.57

$\underline{p} = 11$ UTL = 25 - (1.96)(4.3) = 33.43

$\sigma = 4.3$

$\underline{Z}_{\alpha/2} =$ 1.96 (for two-tailed test from Appendix B-1)

$$\text{Power} = \underline{P}\ [\underline{Z} < \frac{16.57-11}{4.3} \text{ or } \underline{Z} > \frac{33.43-1}{4.3}]$$

$$= \underline{P}\ (\underline{Z} < 1.30 \text{ or } \underline{Z} > 5.22\ )$$

$$= (.9032\ ) + 0 = .9032 \text{ or } 90.3\%\ \text{(Appendix B-1)}$$

If the true proportion is eleven bearings, the power of the test is 90.3%.

The power function given above is a generator for the power curve of a test. This curve represents the ability of a test to detect the existence of a difference when the null hypothesis is not true and is a function of assumed or true mean ($\underline{p}$). Intuitively, one should more easily recognize the difference between the hypothesized value and the true mean as the distance becomes greater. The ideal power curve would be able to detect the difference, no matter how small, with one-hundred percent probability. This does not happen, in reality, when conducting tests of hypothesis yet the power curve does give a relative probability associated with specific values within a population.

There are two factors which affect the power of a test and the curve of its probabilities. The first is the significance level (α). If the alpha risk is set small, the confidence in accepting the null hypothesis ($\underline{H}_0$) when it is true increases. This concurrently, results in an increase in the probability of accepting the null hypothesis when it is, in fact, false (β = type two error). It then becomes apparent that alpha (α) and beta (β) are inversely related in that decreases in one result in increases in the other.

The other factor affecting the power of a test is the sample size. As the sample size is increased, the power of the test ($1-\beta$) is increased. This is the one way by which the probability of committing a type two error (β) can be reduced without affecting the confidence level ($1-\alpha$) of a test.

Examples of power curves are presented in Figure 4.12. These curves represent the proportion ($\underline{p}$) of colored bearings contained within the Sampling Box. The circled point exhibits the power calculated in the last example. Hypothesizing that the sampling box contains twenty-five percent colored bearings ($\underline{p}_o$) and testing for the true proportion ($\underline{p}$), followed by developing the power curves, reveals some noteworthy facts.

Notice the effect of setting alpha (α) at five percent in Curve Two as opposed to Curve Three which is set at ten percent. Curve Two has a higher confidence level than Curve Three however, the power of the test of Curve Two is lower than that of Curve Three.

POWER CURVES: EFFECTS OF SAMPLE SIZE SELECTION AND ALPHA RISKS

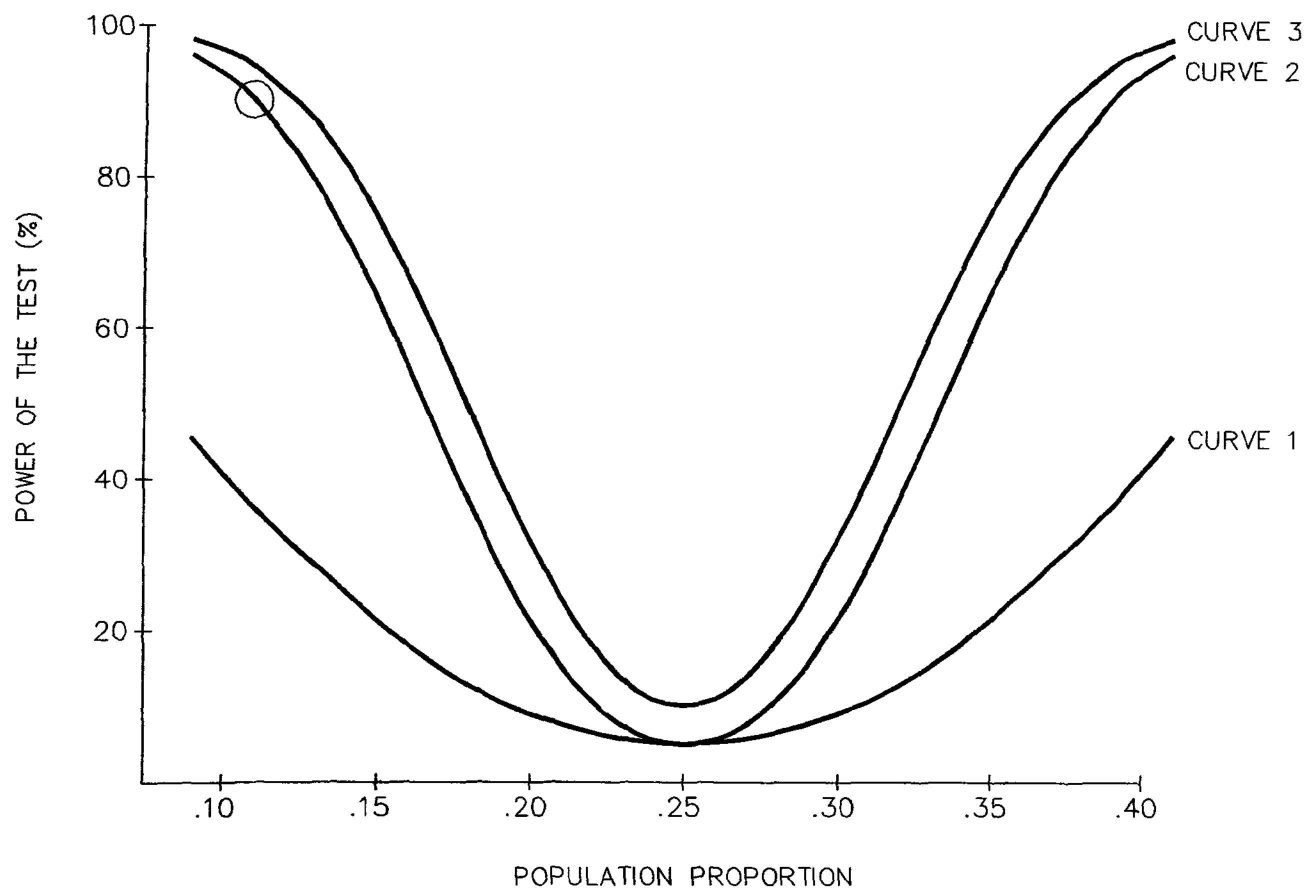


FIGURE 4.12

It is also clear that a one-hundred piece sample (Curve Two) provides greater power for detecting differences between the true and hypothesized proportions than does the twenty-five piece sample (Curve One).

The most important point behind the development of this section has been to show the effects of sample size selection when conducting tests of hypothesis with the Sampling Box. As Figure 4.12 shows, a large sample size yields a greater power of the test. This is a primary reason why the Sampling Box was designed to give variable sample sizes. This is intended to provide students a better grasp of how proper sample size selection can help generate sound decision making practices in manufacturing situations.

OPERATING CHARACTERISTIC CURVE

Another way to demonstrate the relationships affiliated with sample sizes, decision risks and product quality is through the use of an operating characteristic (OC) curve. Also known as a beta (β) curve, this curve shows the probability of accepting a production lot; given the true proportion of defective material. This is also recognizable as the complementary event to the power function; which was given as the probability that the null hypothesis is rejected when it is false (1-β).

All sampling plans possess discernable operational characteristics when samples are taken from their respective populations hence, the name, operating characteristic. Concerning discrete probability, the operating characteristic curve associated with the Poisson distribution is the most widely used by process control engineers. One reason for this is the relative ease by which practitioners can use this distribution to describe sampling relationships. Another important reason, however, is the fact that the Poisson distribution is generally a good approximator to the binomial and hypergeometric distributions. It is for this reason that a description of the operating characteristic curve will be presented in terms of the Poisson random variable.

Given that a discrete random variable is Poisson distributed with previously defined conditions [i.e. $\underline{P}(x) \langle -.10$, $\underline{n} \geq 16$ and $N \geq 10\underline{n}$], the probability mass function for the random variable $\underline{X}$, can be determined by using Equation 4.21; which is presented again for convenience.

$$\underline{P}(\underline{x}) = \frac{e^{-\mu}\,\mu^{-\underline{x}}}{\underline{x}!} \qquad (4.21)$$

As previously noted, this represents the probability of finding exactly x items in a sample of n objects taken from a population. To generate an operating characteristic curve for the Poisson distribution, one must define what the sample size is to be. The next consideration involves the number of defective items (or less) tolerable in a sample. Most practitioners refer to this as the acceptance number (c).

Because the sample size (n) and the fraction defective (p) determine the number of defective items selected, the mean (μ) is taken to be n p. From this, the experimenter can theorize about the true population proportion (p) and manipulate it to determine the probability of obtaining c or less defective items for each true population proportion. Plotting the respective probabilities for each successive treatment of p, for c or less defects, generates an operating characteristic curve.

Example. Suppose it is known that the Sampling Box contains ten percent black bearings and a sampling plan is desired to accept two or less black bearings in each twenty-five piece sample taken from the population. Manipulating the values for each successive value of p, for two or less black bearings using Equation 4.21, will result in a similar fashion to that which follows.

$$P(x) = \frac{e^{-(n\,p)}(n\,p)^{-x}}{x!}$$

where : n = 25 and c = 2

p = .014 or $n\,p$ = (25)(.014) = .35 :

$$P\,(0) = \frac{e^{-(.35\,)}(.35\,)^0}{0!} = .7047$$

$$P\,(1) = \frac{e^{-(.35\,)}(.35\,)^1}{1!} = .2466 \text{ and}$$

$$P\,(2) = \frac{e^{-(.35\,)}(.35\,)^2}{2!} = .0432$$

So, for p = .014; $p(0) + p(1) + p(2)$

= .7047 + .2466 + .0432 = .9945 or 99.5%

This point represents the probability of acceptance (P_a) for selecting two or less black bearings in the sample for a population proportion (p).

By plotting this point and others for successive values of p, an operating characteristic curve of this sampling distribution can be generated. This has been done for the convenience of the reader and is shown in Figure 4.13. The circled point represents the probability calculated in the last example.

Curve One shows the operating characteristic curve associated with a one-hundred piece sampling plan with an acceptance number of two. Curve Two represents a twenty-five piece sampling plan with the

OPERATING CHARACTERISTIC CURVES

C=2

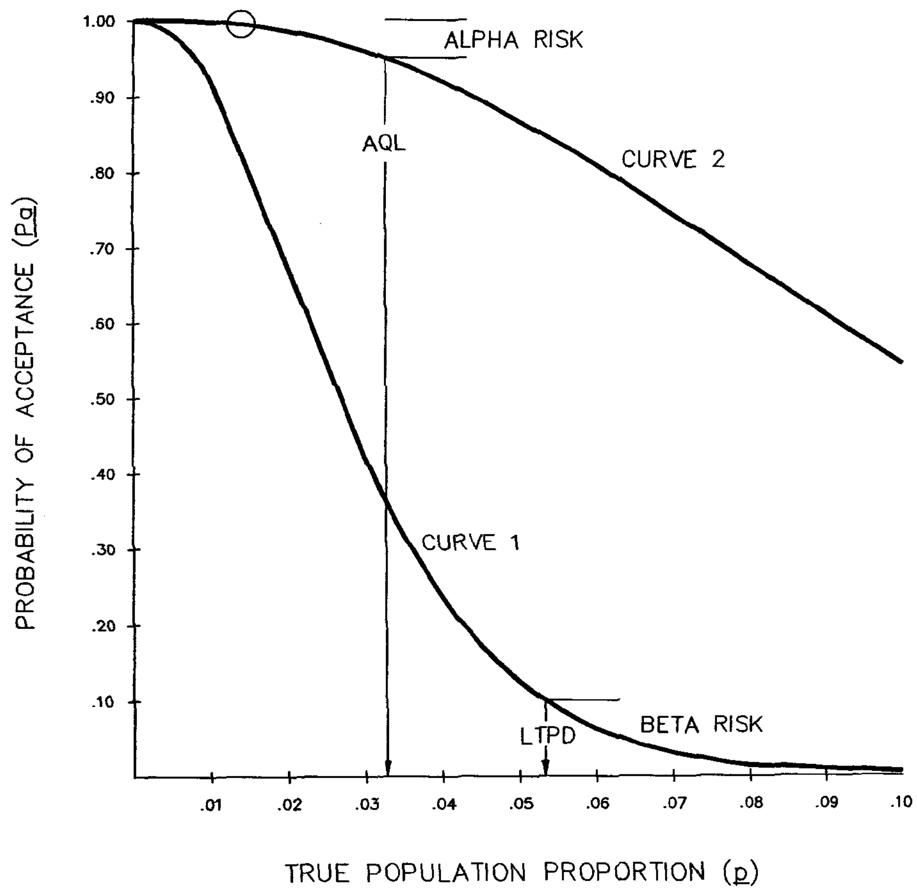


FIGURE 4.13

same acceptance number ($\underline{c} = 2$). Even a casual observation will reveal to the reader a visual grasp of the magnitude of impact that sample size selection has when discriminating between better and worse quality levels.

In Curve Two, the labeled area above and to the right of the curve represents the type one error (α). For these examples, alpha has been assumed to be five percent (other values along this curve could have been selected). The area below and to the left of alpha represents the probability of acceptance ($1-\alpha$) at this level of quality or better. This corresponding value, for fraction defective, is read directly below; along the bottom of the graph. This value of the population proportion ($\underline{p}$) is known as the acceptable quality level (AQL).

The area labeled as the beta (β) risk, in Curve One, represents the type two error, described in earlier sections, and is to the right of the value selected. The corresponding population proportion ($\underline{p}$) to this value is generally called the lot tolerance percent defective (LTPD). The area to the left of this value can be considered the power of the sampling plan ($1-\beta$). The interested reader is encouraged to refer to Appendix C: Definitions of Terms, for more thorough treatment of the terms LTPD and AQL.

Continued description of the performance of these two sampling plans includes the use of an average outgoing quality (AOQ) curve. Intuitively, one recognizes that production lots of very high quality rarely need screening (or sorting) performed on them. Lots of poor quality are likely to be rejected and screened. After sorting, these lots of material become better candidates for acceptance. Marginal lots, conversely, may face less screening (some lots inadvertently pass inspection). It becomes clear that the average outgoing (shipping level) quality of these marginal lots will be worse than lots originally presented at each extreme quality level. At some point, this average quality level will reach a maximum value known as the average outgoing quality limit (AOQL).

The average outgoing quality (AOQ) curve graphically shows the outgoing quality level (screened and unscreened lots combined), given the ingoing population proportion ($\underline{p}$). The AOQ curve is very easy to generate and is based on the following relationship.

$$(\underline{P}_{\underline{a}})(\underline{p}) = AOQ \tag{4.39}$$

Recall from the last example the determination of the probability of accepting a sample of twenty-five pieces with an acceptance number of two black bearings; where the true population proportion ($\underline{p}$) was 1.4%. Using Equation 4.39, determine the average outgoing quality level for these values:

$\underline{P}_a = .9945$

$\underline{P} = .014$

Substituting these values into Equation 4.39 and calculating the AOQ leaves:

(.9945)(.014) = .013923 or 1.4%.

This means that an average lot of this quality ($\underline{p}$ = .014) will contain 1.4% defective product at the point of shipping. This value can be seen as the circled point in Figure 4.14, Curve One.

The points labeled AOQL on Curve One and Curve Two represent the worst levels of quality shipped under these sampling plans; given the ingoing quality ($\underline{p}$) listed at the bottom of the graph. Again, it appears that the one-hundred piece sampling plan offers the greatest degree of protection against loss.

Table 4.7 summarizes some of the results obtainable, relating to both the twenty-five and one-hundred piece samples taken from the Sampling Box. It becomes increasingly clear that the one-hundred piece sampling plan out-performs the twenty-five piece sampling plan.

AOQ CURVES

C=2

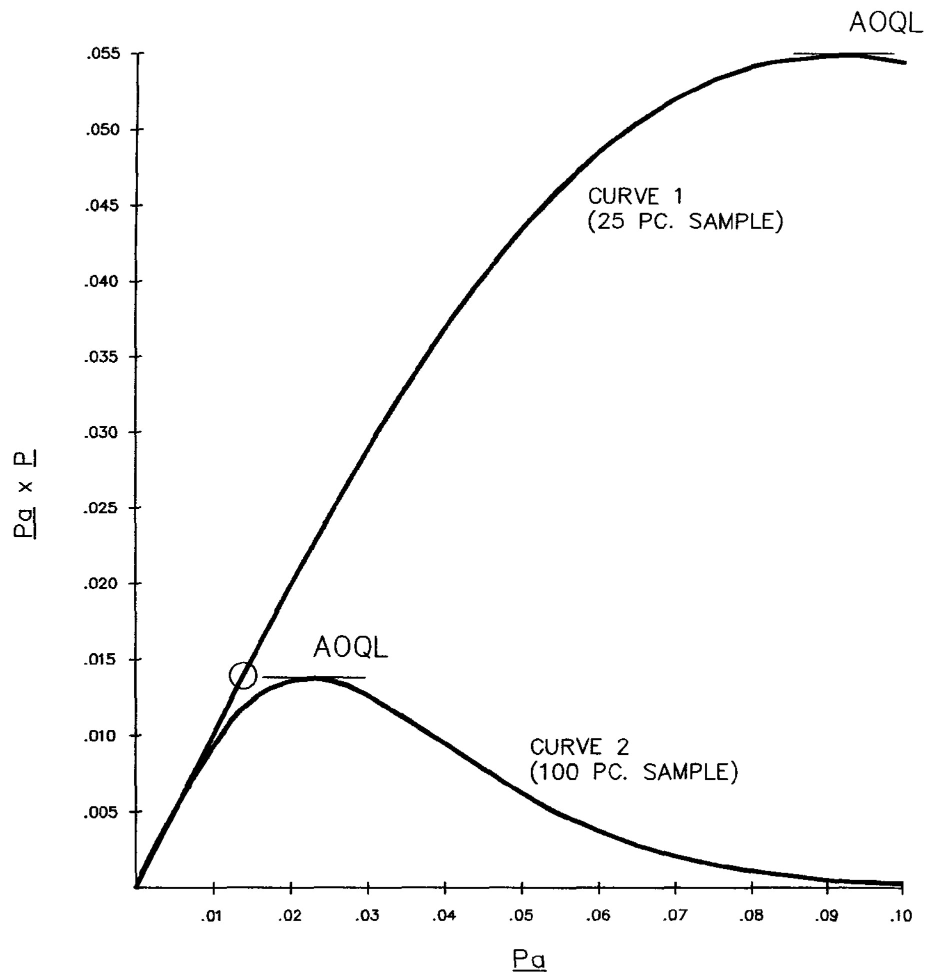


FIGURE 4.14

COMPARISON OF 25 AND 100 PIECE SAMPLING PLANS BASED ON OPERATING CHARACTERISTIC CURVES

Alpha = .05
Beta = .10

Twenty-five piece sample	One-hundred piece sample
<u>C</u> = 2	<u>C</u> = 2
AQL = 3.3%	AQL = .4%
AOQL = 5.5%	AOQL = 1.4%
LTPD = 21.2%	LTPD = 5.3%

Table 4.7

Understanding the concepts associated with decision risks and sampling plan performance is a major part of what manufacturing process control is all about. The previous examples illustrate a few of the possible relationships, relating to acceptance sampling, that the Sampling Box can help simulate in the classroom. The ability to conceptualize many of these relationships was a primary goal, desired by this researcher, when development of the Sampling Box was first considered. With a little imagination and creativity; a good instructor, such as Mr. Don Creger, should find the Sampling Box a useful aid in teaching these concepts to students.

SAMPLE SIZE SELECTION FOR CONTROL CHARTING

Sample size selection for control charting purposes requires careful consideration. Sample sizes which are too large impose excessive inspection costs and degrees of accuracy unnecessarily stringent for each application. Underestimation of performance is likely to result from small sizes which fail to detect true process capability. In fact; when sample sizes are too small, the presence of just one defect during sampling will indicate a lack of control. In this particular case, even a well-trained operator will attempt to correct a process which may or may not need adjustment and will likely over or under react to an underestimated process.

Concerning the ideal level of control chart sensitivity, it has been stated that "The sample should be large enough so that at least nine times out of ten one or more defectives will be found."[3] A solution to this requirement is straightforward with the use of the following formula.

$$\underline{n}\,\bar{\underline{p}} - 2\sqrt{\underline{n}\,\underline{p}\,(1-\bar{\underline{p}})} = 1 \tag{4.40}$$

Isolating the value for n it now reads:

$$\sqrt{\underline{n}} = \frac{\sqrt{2-\bar{\underline{p}}} + \sqrt{1-\bar{\underline{p}}}}{\sqrt{\bar{\underline{p}}}} \tag{4.41}$$

Notice that the above expression is merely a manipulation to the process average minus two sample standard deviation units and is equal to one. The reason the expression is set equal to one is because this is the desired number of defects obtained in each sample. The purpose for using two standard deviation units is because the objective is to find defective units nine out of ten times (or ninety percent probability) and

[3]William B. Rice, Control Charts in Factory Management (New York: John Wiley & Sons, Inc., 1974), p.82.

the two standard deviation limit ($\alpha \approx .05$) actually exceeds the confidence level desired for defective units in this case.

Example. Suppose that a process is known to operate at a defect rate of ten percent ($\bar{p} = .10$) and we wish to establish a control chart which will detect at least one defective unit nine times out of ten. Equation 4.41 can be used to determine the sample size necessary to satisfy this minimum requirement in establishing the lower control limit as follows:

$$\sqrt{n} = \frac{\sqrt{2-.10} + \sqrt{1-.10}}{\sqrt{.10}} = 7.358+.$$

Solving for n yields 54.2 or 55 pieces. Because sample sizes can only take on integer values, this value is rounded up to fifty-five as the minimum requirement.

An example of a control chart utilizing the previously illustrated technique is presented in Figure 4.15. This particular chart is known as an $n\ p$ chart and is used to measure the number of defective items in a subgroup of items taken from a population. In this particular case, the expected value or the mean (p) is denoted by $n\ \bar{p}$ which is the number of items in the sample times the average proportion ($\bar{p}$); based on historical data. This is represented on the graph as a solid line, at the expected value, across the length of the chart. The calculation of $n\ \bar{p}$

np DATA SHEET

Plant

Department no.

Machine no.

Pcs. per hr. 55

Subgroup size 55

Sample size 1,375

Part Name ______ Part No. ______

Oper. No. & Description ______

No.	Number Defective	Fraction Defective	Reasons for Reject					Remarks	DATE	TIME	OPER	INSP
1	9											
2	6											
3	4											
4	4											
5	6											
6	5											
7	3											
8	6											
9	5											
10	3											
11	8											
12	4											
13	6											
14	10											
15	5											
16	8											
17	4											
18	5											
19	2											
20	5											
21	7											
22	6											
23	5											
24	7											
25	4											
Σ	137											
%	0.100											

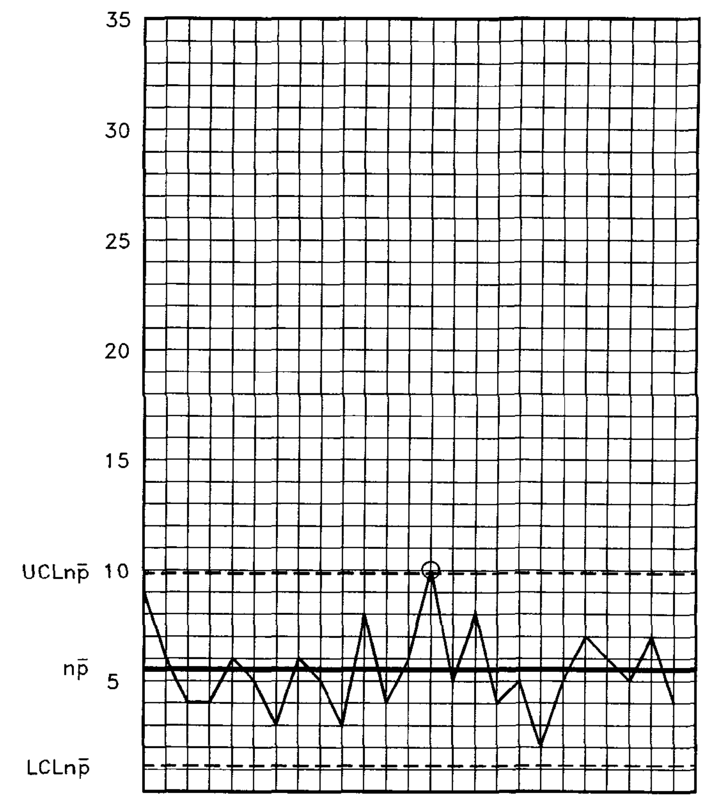


Figure 4.15

and the control limits for this chart are based on Equation 4.21 and are presented below.

$\underline{n} \quad = 55$

$\underline{n}\ \underline{p} = .10$ (average of past proportion)

$\underline{n}\ \bar{\underline{p}} = (55)(.10)$

$$\text{Lower control limit } (LCL_{\underline{n}\,\bar{\underline{p}}}) = (55)(.10) - 2\sqrt{(55)(.10)(.99)}$$
$$= 1.05 \text{ or } 1$$
$$\text{Upper control limit } (UCL_{\underline{n}\,\bar{\underline{p}}}) = (55)(.10) + 2\sqrt{(55)(.10)(.90)}$$
$$= 9.95 \text{ or } 10.$$

The lower and upper control limits on the chart are shown as the broken (or dashed) lines and represent the points at which a state of lack-of control is likely to exist. Note that the circled point on the graph was the one sample that exhibited this condition. It also appears evident that the goal of finding at least one defective item at least once in ten trials has been accomplished. In fact, the smallest number detected was two.

The reader will notice that the sample size in this case was fifty-five pieces. The Sampling Box was designed to deliver four classes of twenty-five pieces. For this experiment, three classes of twenty-five bearings had to be selected however, only the first fifty-five units were counted; the remaining twenty bearings were simply ignored. Figure 4.16 shows the number of bearings counted as those which are

SAMPLE SIZE SELECTION FROM SAMPLING BOX:
N = 55 PIECES

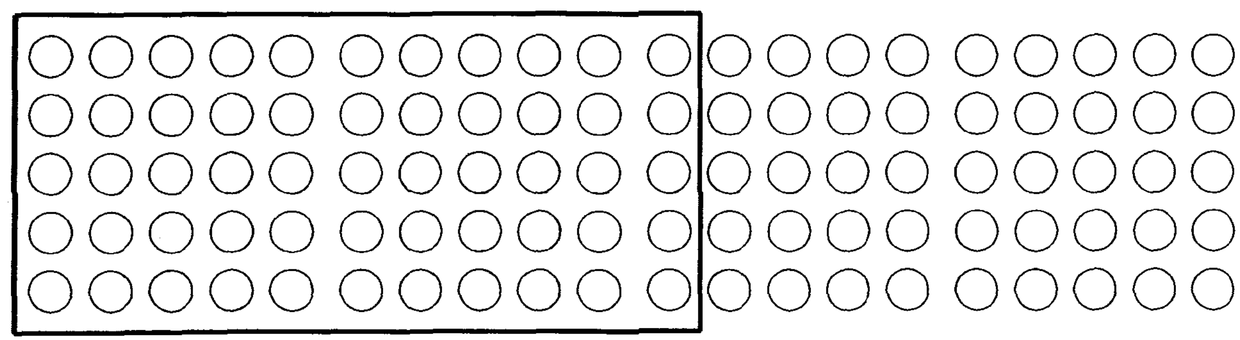


FIGURE 4.16

contained within the bounding region. When required sample sizes do not match with those provided by the Sampling Box, this practice is recommended by this researcher.

It may now be readily apparent that the previous example considers the lower control limit and the minimum number of defects detected in the sample. These are important control chart characteristics that should not be overlooked because they help prevent underestimation of process performance and define minimum sample size requirements.

Another means of determining sample size requirements for control charting purposes involves consideration of the upper control limit as well as the lower control limit. This approach focuses on maintaining a desired level of quality while providing a degree of sensitivity necessary to detect shifts in process performance. For example, it may be desirable to recognize process changes which result in deterioration of product integrity and increased waste. In such cases, a maximum average acceptable defect rate may serve as partial criterion for sample size selection.

Example. Consider a process which is known to operate at a ten percent average defect rate. If the maximum tolerable rate is twenty-five percent and a fifty percent chance of detecting this degree of shift on the first sample is desired with greater than ninety-nine percent confidence, the sample size necessary to satisfy these requirements can be determined. In mathematical terms this means $\bar{p} + 3\sigma_{\bar{p}} = 25\%$.

Using the expression:

$$\underline{\bar{p}} + 3\sqrt{\frac{\underline{\bar{p}}(1-\underline{\bar{p}})}{\underline{n}}} = \underline{L} \tag{4.42}$$

and solving for $\underline{n}$ yields:

$$\underline{n} = \frac{9\underline{\bar{p}}(1-\underline{\bar{p}})}{(\underline{L}-\underline{\bar{p}})^2} \tag{4.43}$$

where: $\underline{L}$ is the process limit. Substituting the process values into Equation 4.43 results in the following:

$$\underline{n} = \frac{9(.10)(1-.10)}{(.25-.10)^2} = 36$$

The minimum sample size required is 36 units.

This information can now be used to establish a control chart for continued sampling of this process. The chart best used for this application is the $\underline{p}$ chart. This particular chart measures the proportion of defective items from each subgroup of items taken as a sample. Calculation of the process average ($\bar{p}$) and the respective control limits for this example, from Equation 4.42, are as follows:

$n = 36$

$p = 0.10$ (based on past performance)

$$\text{Lower control limit } (LCL_{\bar{p}}) = (.10) - 3\sqrt{\frac{(.10)(.90)}{36}} = -.05 \text{ or } .00$$

$$\text{Upper control limit } (UCL_{\bar{p}}) = (.10) + 3\sqrt{\frac{(.10)(.90)}{36}} = .25.$$

The resulting control chart is shown in Figure 4.17. The data taken during experimentation with the Sampling Box appear to be truly random in nature and are well within the control limits. Notice that the lower control limit is zero. Although the calculated value was slightly below zero, one can never obtain less than no defective items in a sample. This is the commonly accepted practice in manufacturing settings and is the reason why it was set at this value.

Another, slightly different , approach toward achieving a specific level of protection with a sampling scheme considers the degree of quality as well as the confidence level and utilizes tabular values of the standard normal distribution as an approximation to the binomial distribution in terms of standard Z-score units. Assuming that the

p DATA SHEET

Plant

Department no.	Part Name ______ Part No. ______
Machine no.	Oper. No. & Description ______
Pcs. per hr.	
Subgroup size 36	
Sample size 900	

No.	Number Defective	Fraction Defective	Reasons for Reject					Remarks	DATE	TIME	OPER	INSP
1	2	0.056										
2	4	0.111										
3	6	0.167										
4	3	0.083										
5	4	0.111										
6	5	0.139										
7	2	0.056										
8	4	0.111										
9	1	0.028										
10	6	0.167										
11	3	0.083										
12	4	0.111										
13	4	0.111										
14	3	0.083										
15	1	0.028										
16	5	0.139										
17	1	0.028										
18	6	0.167										
19	4	0.111										
20	3	0.083										
21	3	0.083										
22	2	0.056										
23	5	0.139										
24	3	0.083										
25	4	0.111										
Σ	88	2.445										
%	0.098	0.098										

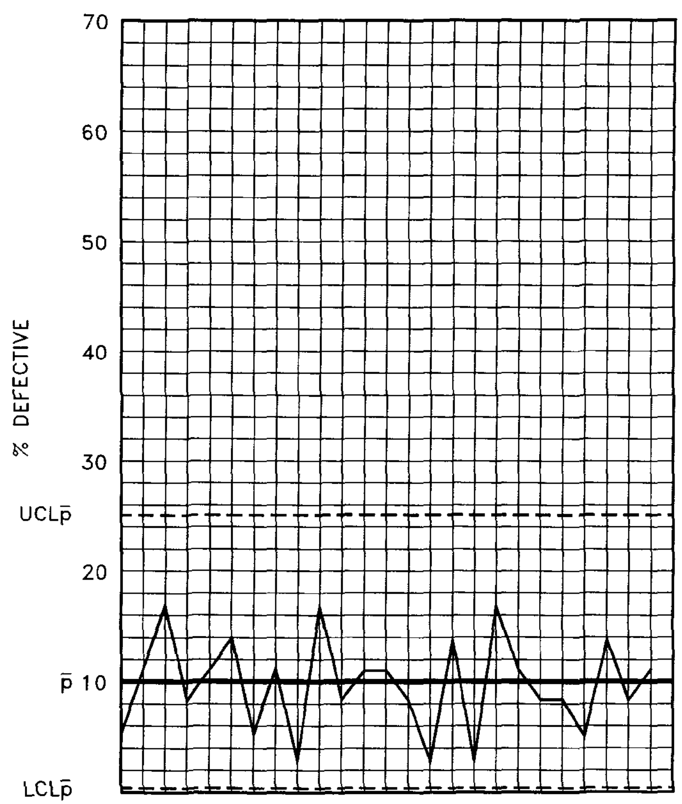


Figure 4.17

population random variable is normally distributed, an estimate of the population proportion (p) can be determined to within d units of the true value at a given level of confidence as shown below:

$$n = Z^2_{\frac{\alpha}{2}} \frac{\bar{p}(1-\bar{p})}{d^2} \tag{4.44}$$

where:

$\bar{p}$ = an estimate of the true population proportion (p)

d = the number of units (or difference) from the true population proportion

$Z_{\alpha/2}$ = the standard normaldistribution value for a given level of confidence

Example. If the previously defined process operating at ten percent defects is to be controlled to within fifteen units (or d = .15) at a ninety-five percent confidence level, use of Equation 4.44 and Appendix B-1 results in the following:

$$n = (1.96)^2 \frac{(.10)(1-.10)}{.15^2} = 15.4 \ or \ 16 \ pcs.$$

Thus, sixteen pieces is the minimum number required to satisfy these conditions.

Having determined the sample size requirements for this sampling plan, the practicioner can now establish a control chart with action limits as determined by using Equation 4.45 presented below.

$$\bar{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (4.45)$$

It is known that:

$\bar{p} = .10$

$n = 16$

$\alpha = .05$ (arbitrarily set at .05)

$Z_{\alpha/2} = 1.96$ (Appendix B-1)

After substitution into Equation 4.45 and calculating the control limits, they now become:

$$LCL_{\bar{p}} = .10 - (1.96)\sqrt{\frac{(.10)(.90)}{16}} = .-0.047 \text{ or } .00$$

$$UCL_{\bar{p}} = .10 + (1.96)\sqrt{\frac{(.10)(.90)}{16}} = .247 \text{ or } .25$$

(as prescribed in the example).

The control chart representing these data is shown in Figure 4.18. The data collected from the Sampling Box by this researcher appear to be randomly distributed, which is desirable. Notice the number of samples taken possessing no black bearings. On the control

p DATA SHEET

Plant		
Department no.		Part Name ______ Part No. ______
Machine no.		Oper. No. & Description ______
Pcs. per hr.		
Subgroup size	16	
Sample size	400	

No.	Number Defective	Fraction Defective	Reasons for Reject					Remarks	DATE	TIME	OPER	INSP
1	2	0.1250										
2	1	0.0625										
3	2	0.1250										
4	0	0.0000										
5	2	0.1250										
6	0	0.0000										
7	1	0.0625										
8	2	0.1250										
9	1	0.0625										
10	3	0.1875										
11	2	0.1250										
12	4	0.2500										
13	0	0.0000										
14	1	0.0625										
15	3	0.1875										
16	2	0.1250										
17	1	0.0625										
18	0	0.0000										
19	1	0.0625										
20	3	0.1875										
21	2	0.1250										
22	1	0.0625										
23	2	0.1250										
24	0	0.0000										
25	1	0.0625										
Σ	37	2.3125										
%	0.9250	0.9250										

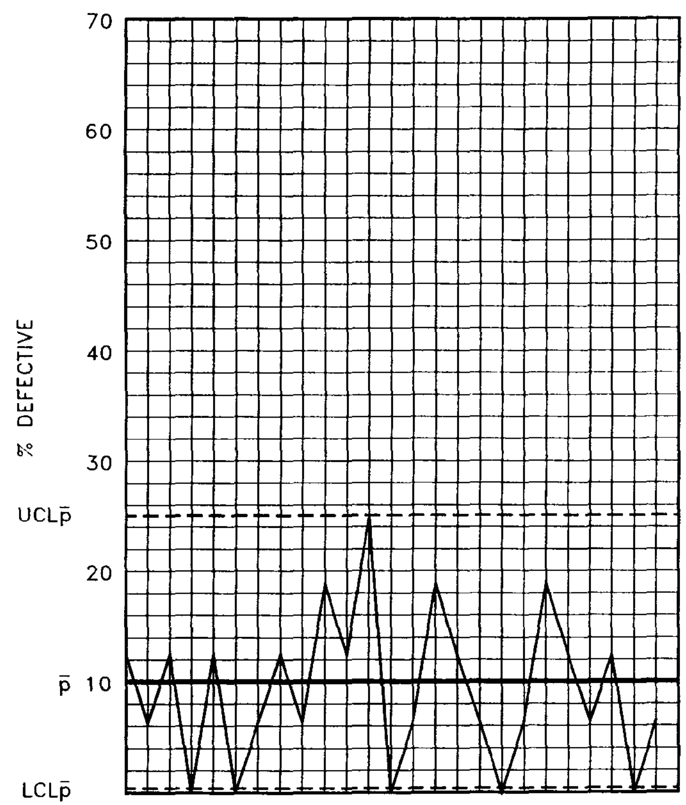


Figure 4.18

chart, this may appear to represent out-of-control situations when, in fact, they may not exist.

The reader may recognize the lack of symmetry between the respective control limits and the process average ($\bar{p}$). The reason for this is because the lower control limit had to be compressed to conform to the sampling distribution. This results from selecting sample sizes which are too small to adequately reflect the variability of the process under investigation.

Although the upper control limit satisfies the prescribed sensitivity to detect process shifts up to twenty-five percent, it can be seen that the lower control limit proves to be unsatisfactory. When using Equation 4.45 to determine sample size requirements, it is well advised to consider the effects of setting alpha (α) and consequently, $Z_{\alpha/2}$.

It may be apparent to the reader that Equation 4.44 is similar to Equation 4.43 in that the approach toward determining a sample size is essentially the same. The primary difference between the two methods is that Equation 4.44 more easily accommodates the user in establishing the level of confidence ($Z_{\alpha/2}$) than does Equation 4.43. Equation 4.43 fixes this value ($Z_{\alpha/2}$= 3) and concentrates primarily on the relationship between the true population mean and the point of detection in terms of the maximum tolerable process average (L).

Notice that the sample sizes determined by the two methods are different [36 (Equation 4.43) $\rangle$ 16 (Equation4.44)]. The reason for this difference is due solely to the confidence level desired. It is, therefore, necessary that the practitioner consider very carefully any decisions relating to sample size selection. Moreover, when the requirements of Equations 4.41, 4.43 and/or 4.44 must be met; it is generally accepted that the larger sample size be used to assure the greatest protection against loss; when it is economically feasible.

The previous examples show that proper sample size selection can help provide an ideal degree of protection against loss from control charting and acceptance sampling schemes. This is a primary reason why the Sampling Box was designed and built to provide multiple sample size selections.

The experience of this researcher is that, altogether too often, sample size selection is not adequately addressed in manufacturing settings. Part of the reason for this is because many students have not been thoroughly presented the fundamental requisites to proper sample size selection. Student experiments with the Sampling Box should provide a testament to the validity of the logic presented in this section and further aid in understanding the nature of process variability and how proper sampling can help measure it.

CHAPTER FIVE

EDUCATOR SURVEY

Throughout the majority of this project, several well-known process control educators from northern Illinois provided this researcher with a wealth of advice relating to sampling devices and their uses. The input from their teaching experience and knowledge of training aids resulted in the development of a sampling device which would benefit both the student and the instructor. The advice they provided also included applications for which the device was intended.

In order to adequately test the worthiness of the final product and the potential for its use, three educators with considerable experience in teaching corporate-based, in-house training programs, seminars and workshops and/or college-level programs in academic institutions were selected to complete an educator survey.

Each participant was given the Sampling Box, developed by this researcher, to use for as long as was needed to weigh the merits of the device, compare it with other devices and complete a survey related to training aids, course content and student learning. The survey consisted of fourteen questions; focusing on course content, applicable training devices and characteristics desired of training aids. The results of this survey are presented in Appendix E (Educator Survey). However, the paragraphs which follow highlight some of the key points ascertained through the survey.

Each of the respondents in the educator survey stated that they have used (or would use) most of the training devices presented in this paper. In order of importance to the instructor; the devices receiving the highest priority ranking were Sampling Box, Random Dice, Quincunx and Sampling Bowl, respectively. These were followed by Roman Catapult and Run Demonstrator.

Concerning course content, each respondent agreed that training devices would be particularly useful for assisting in teaching probability theory, acceptance sampling, sampling variability, control charting and operating characteristics of sampling distributions. Other topics were noted however, these were the most commonly supported with the use of training aids.

Early in the developmental stages of this project, it was necessary to determine the characteristics associated with sampling devices which would be most desired by an instructor. When prioritizing characteristics, the respondents stated that a reasonably compact, portable and durable device, which was quick and easy to use, was of high priority. Moreover, it was desired that devices effectively aid in teaching the merits of sample size selection, minimize the potential for sampling bias and allow for efficient replacement of sample elements back into the population without loss yet be flexible in delivering variable sample sizes.

Having prioritized which characteristics were most desired of training devices, the respondents were then asked to compare the merits of a sampling box to those of a sampling bowl. The results indicated that a sampling box is more compact, portable and easy to use than a sampling bowl and is at least as durable and lightweight as a sampling bowl. The responses given indicate that the speed and ease of sample taking and counting with a sampling box may be perceived as more efficient than with a sampling bowl.

Each device appeared to have equal ratings on replacement of sample elements back into the general population and on maintenance of population parameters. Relative to population maintenance, a sampling box has a sealed population which prevents loss of sample elements. Whereas, a sampling bowl is an open container which may more readily relinquish sample elements to loss. Changing population parameters with a sampling bowl is quicker and easier than with a hermetically-sealed sampling box. Another advantage of the sampling bowl can be seen in its' ability to deliver more flexible sample sizes.

A comparison of a sampling box to a sampling bowl was performed relative to teaching course-related content. The responses implied that a sampling box could help improve productivity of student participation in classroom exercises; aiding students to more rapidly disseminate course content and may better foster student commitment to principles promoted by the instructor than with the use of a sampling bowl.

When asked if the use of a particular device would better help clarify topics which were difficult for students to grasp or if students would learn more with a particular device, no real preference was given to the sampling bowl or sampling box. There was, however, an overall preference for using a sampling box when considering the ideal classroom situation.

The participation of the educators and the input they provided in the survey was considered a valuable part of this research project. This was particularly true because they each took the time to fully experiment with the Sampling Box developed by this researcher and promptly reported, with candor, their responses. The concern, however, is the fact that many logistical constraints limited the number of participants to three and, with a very limited sample size, little can be conclusively drawn from the survey responses.

CHAPTER SIX

SUMMARY

Attempts to improve the efficiency by which students assimilate course content, relative to manufacturing process control, have included the commercial development of training devices for simulating manufacturing processes and demonstrating sampling relationships. Among the mechanical devices widely marketed for commercial use are Random Dice, Quincunx, Catapult, Sampling Bowl (or Box) and Run Demonstrator. Each of these devices are offered in a variety of different forms. Depending upon manufacturer or marketing firm, these devices can cost as little as $50.00 for Random Dice or even over $1,000.00 for a highly sophisticated Quincunx.

These devices are increasingly becoming popular for use by academic institutions, consulting firms, in-house manufacturing trainers and others for teaching nearly any process control topic. Western Illinois University also uses a training device for teaching Manufacturing Process Control, IE&T 345 in the Industrial Education and Technology Department. By using a Sampling Bowl, the students are better able to grasp the nature of such topics as probability distribution forms, hypothesis testing, sampling techniques, control charting practices, decision making and risk assessment methodology.

Although sampling devices vary widely as to utility and design configuration, some of the most desirable characteristics to consider when selecting training aids may be speed and ease of use, compactness, lightweight design, portability, flexibility in application, etc... The most important criterion however, is selection of a device (or devices) which most effectively aids in transformation of course related content into a format easily applicable to the student by demonstrating the nature of manufacturing processes and how they may be controlled.

The problem of this research was to develop sampling device which was highly portable, lightweight, compact and hermetically sealed. This necessitated an evaluation of current sampling device technology. A literature review was conducted and an analysis of research data lead this researcher to conclude that a Sampling Box could serve the needs of Western Illinois University in Teaching Manufacturing Process Control. This choice was based on recognition of current utility needs as well as convenience of use in expeditiously teaching process control methodology to students.

In order to test these assertions, developmental plans were prepared and professional advice was solicited from several highly-respected process control educators indigenous to the northern Illinois region. Their input was graciously incorporated into the development of a Sampling Box and the material included in the text manual.

Because this researcher believed modifications to existing Sampling Box technology were possible, a Sampling Box was designed and constructed to include, what this developer refers to as a variable, quatropartite sample delivery system. This development improves sample size flexibility potential and minimizes the number of population elements exposed for view. This latter point is intended to reduce the possibility for sampling bias. Concurrent relevance to the needs of the Industrial Education and Technology Department of Western Illinois University was prepence and adhered to. Moreover, this device improves ease of portability, efficiency of use and the maintenance of population parameter requirements as specified in the proposal.

Provision of an applications manual has been included to highlight some of the potential uses for the Sampling Box. Material contained within the manual often reflects rigorous treatment of topics beyond the scope of the class in question. This has been done in order to prove the reasoning behind the development of the Sampling Box as configured. Although the needs of the students have been carefully considered, the manual itself was prepared for the instructor who must necessarily decide the fate of the Sampling Box and its use.

CHAPTER SEVEN

RECOMMENDATIONS

Constant query and contemplation into the nature of how the Sampling Box would best be configured and how it could serve the needs of the University were at the very heart of this entire project. Equally pervasive were the incessant comparisons of the Sampling Bowl to the Sampling Box by this researcher. The amassed collation relative to these respective devices has unveiled a number of facts which this author would be remiss in not presenting.

As with the Sampling Bowl, the Sampling Box is very well suited for demonstrating concepts relative to the following examples.

1) Point estimates and confidence interval determinations for most sampling distributions.
2) Acceptance sampling and control charting practices.
3) Risk assessment and decision making.
4) Sample size selection and sampling errors.
5) Tests of hypothesis.
6) Operating characteristics.
7) Power function.

While these functions (and others) can be satisfactorily demonstrated by both devices, neither device has been designed to recapitulate conditional/compound probability theory, including Baye's Theorem, nor are they particularly well suited for teaching concepts associated with the hypergeometric distribution. As shown in the facilitator's applications manual, large population and sample sizes require calculations so unwieldy they will certainly seem insurmountable to the typical student.

This author does not recommend that one device be used at the exclusion of the other. In fact, there may be occasions where both devices can be used in comparison with each other. There are numerous applications where this will hold true and for these situations; the use of both devices is suggested.

The Sampling Box is highly portable and was designed so that students may freely pass the device back and forth however, the Sampling Bowl is larger and more visible to students in a large class when an instructor performs demonstrations for the simultaneous benefit of all audience members.

While the Sampling Box can more quickly deliver standard sample sizes, the Sampling Bowl has greater flexibility in sample size selection capability. The Sampling Bowl also provides much greater color combination possibilities than does the Sampling Box. This is also true for commercially developed Sampling Boxes; which typically offer up to seven different color combinations in varying percentages. Due to the limitations in sufficiently contrasting, plating coloring possibilities; this limits the potential for multiple comparisons of sample items.

As the number of colored items in a population increases to as much as fifty percent, the binomial distribution more closely approximates the normal distribution. Conversely, as the number of colored items in a binomially distributed population decreases, the population more closely approximates the Poisson distribution. In the former case, the Sampling Box may be a better device to use because of its smaller population size and somewhat larger proportion of colored items. This is more ideal for demonstrations involving the normal or binomial distributions. In the latter case, the Sampling Bowl may be a better choice because of its large population size and more fractionable proportion of colored items. In either case, both devices can be used (exercising prudence) to depict these relationships.

Although a myriad of reasoning can be brought forth, these examples provide sufficient evidence that preponderance does not necessarily weigh favorably for one device at the exclusion of the other for both devices were designed to perform the same basic functions. When issues of portability, cycle times between sample taking, standardization of student test results and reduced sampling bias are of paramount concern; this researcher recommends use of the Sampling Box.

As previously noted, material presented in the facilitator's applications chapter often goes well beyond the scope of the class. This was performed to display some of rationale behind development of the Sampling Box. For the sake of teaching Manufacturing Process Control, IE&T 345; the Sampling Box can be used to help demonstrate relationships relative to the concepts presented below.

1) Data collection methods.
2) Histograms, Pareto diagrams, cause and effect diagrams, etc...
3) Description of statistical concepts.
4) Development of control charts and analysis of data.
5) Establishment of rational subgroups; including sample sizes, sampling frequency and conditions for sampling.
6) Generation of capability indices such as CP, C_{PK} or PC ratio.
7) Acceptance sampling, decision making and risk assessment for

AQL or Mil.Std./OSE based sampling plans.

8) Deming's Red Bead experiment.

The previous list includes course content directly relevant to teaching Manufacturing Process Control, IE&T 345 at Western Illinois University. Mastering these concepts is required, as specified in the course syllabus, and is well within the capabilities of the Sampling Box. Satisfaction of these requirements with ease and efficiency was the primary reason for developing the Sampling Box. It is therefore, recommended that this device be used to demonstrate these types of relationships.

Considering the manufacturing resources, monetary assets and materials at the disposal of this developer; the resultant, Sampling Box should prove useful to Western Illinois University. In the event that an effigy of the Sampling Box may even be required, a number of improvements are possible.

This developer would place increased salience on the choice of materials for construction. Ideally; lucite, nylon or other hard yet fracture resistant materials would make good candidates for frame construction and would most certainly be appropriate for plate development. For mass production, these materials would be economically frugal choices and have very good properties for injection or centrifugal molding. They are also lightweight solutions to the characteristics desired by this developer.

Another avenue for modification involves the current use of ball-bearings; of which the sampling population is comprised. Their use adds greatly to the overall weight of the Sampling Box and their function would best be served by precision molded and polished lucite beads or other materials possessing similar properties. Whereas, the ball-bearings are plated in only three colors, lucite (or other) beads can be obtained in a variety of solid-molded colors and would significantly improve sampling population variability comparisons. They are not succeptable to color fading or wear as are the plated bearings. The inevitable consternation over replating or replacement of plated bearings will likely denigrate the relatively pristine qualities of the Sampling Box and can render this device virtually useless.

Despite the previously-noted caveats and areas for improvement, the Sampling Box is designed and built to provide the user many years of felicitous use. Should there ever be need for maintenance or repairs, the previously illustrated part drawings can serve as a basis for construction or improvement.

CHAPTER EIGHT

CONCLUSIONS

Western Illinois University currently uses a Sampling Bowl as a training device in the Industrial Education and Technology Department for teaching Manufacturing Process Control, IE&T 345. Because this researcher believed that an alternative training device could better match the needs of both the University and its students, the problem of this research was to determine if such device could be obtained or developed to serve as an alternative to the Sampling Bowl.

The objective was to develop a device which was highly portable, light weight and compact. It was further required to obtain a training device which inherently possessed a secure population constituency to avoid loss of sample elements and minimize the potential for classroom disruption as well as help standardize population parameters for testing and comparison purposes. Another objective was to improve the efficiency of sample extraction, counting and replacement of sample elements back into the general population.

After analysis of existing technology and review of literature, pertaining to process control devices, the Sampling Box was selected as a possible alternative device. Currently marketed Sampling Box technology matches well with the needs of the University, however, a possible modification to existing designs ultimately lead to what this developer refers to as a variable, quadropartite sample delivery system. This design configuration provides sample size flexibility which matches or exceeds any currently marketed Sampling Box in one easy step and maintains the greatest potential for secrecy of extraneous population elements to minimize sampling or reporting bias.

The Sampling Box designed by this developer is reasonably lightweight, compact and highly portable. This device is hermetically sealed, maintenance of population parameters are assured and the potential for classroom disruption has been reduced. By merely depressing a lever, variable sample sizes can be rapidly delivered, counted and replaced back into the general population to improve the efficiency of classroom experiments.

A facilitator's manual was developed which serves as Chapter IV of this project and highlights some of the extreme capabilities of the Sampling Box through somewhat rigorous treatment of subject matter. Development of the manual and the device was subject to scrutiny by several leading, regional process control educators to assure product integrity and applicability of the device to its intended purpose.

Although the Sampling Box matches all of the original expectations of this researcher, it is not necessarily intended to replace the Sampling Bowl. These devices can be used interchangeably and can even be used simultaneously as a basis for comparisons. The Sampling Box is built to be rugged and durable, complements course-specific content very well and should serve the needs of the University and its students for many years to come.

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APPENDIX A

TABLE OF EQUATIONS

TABLE OF EQUATIONS

$$\mu \approx \underline{\overline{X}} = \frac{\sum_{\underline{i}=1}^{\underline{n}} (\underline{x}_{\underline{i}})}{\underline{n}} \tag{4.1}$$

$$\sigma^2 \approx \underline{s}^2 = \sum_{\underline{i}=1}^{\underline{n}} \frac{(\underline{x}_{\underline{i}} - \underline{\overline{X}})^2}{\underline{n}-1} \tag{4.2}$$

$$\sigma \approx \underline{s}^2 = \sqrt{\underline{s}^2} \tag{4.3}$$

$$\underline{f}(\underline{y}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{4.4}$$

$$\underline{Z} = \frac{\underline{x}_{\underline{i}} - \mu}{\sigma} \tag{4.5}$$

$$\underline{p} = \frac{\text{number of occurances}}{\text{sample size}} \tag{4.6}$$

$$\underline{P}(\underline{x} \text{ occurs in } \underline{n} \text{ trials}) = C_{\underline{x}}^{\underline{n}}\ \underline{p}^{\underline{x}}\ (1 - \underline{p})^{\underline{n}-\underline{x}} \tag{4.7}$$

$$E(X) = n\,p \tag{4.8}$$

$$Var(X) = n\,p(1-p) \tag{4.9}$$

$$\sigma_x = \sqrt{np(1-p)} \tag{4.10}$$

$$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \tag{4.11}$$

$$LCLp = \frac{2r-1-Z^2_{\alpha/2}-Z_{\alpha/2}\sqrt{[\frac{(2r-1)(2n-2r+1)}{n}]+Z^2_{\alpha/2}}}{2(n+Z^2_{\alpha/2})} \tag{4.12}$$

$$UCLp = \frac{2r+1+Z^2_{\alpha/2}+Z_{\alpha/2}\sqrt{[\frac{(2r+1)(2n-2r-1)}{n}]+Z^2_{\alpha/2}}}{2(n+Z^2_{\alpha/2})} \tag{4.13}$$

$$P(x) = \frac{C^{N-M}_{n-x}\,C^{M}_{x}}{C^{N}_{n}} \tag{4.14}$$

$$E(x) = \frac{(n)(M)}{N} \tag{4.15}$$

$$\underline{V}(\underline{X}) = \frac{\underline{N}-\underline{n}}{\underline{N}-1}[\underline{n}(\frac{\underline{M}}{\underline{N}})(1 - \frac{\underline{M}}{\underline{N}})] \tag{4.16}$$

$$\underline{p} \pm \underline{Z}_{\alpha/2}\sqrt{\frac{\underline{p}(1-\underline{p})}{\underline{n}}(1-\frac{\underline{n}}{\underline{N}})} \tag{4.17}$$

$$\underline{Y} = \frac{e^{\frac{-\chi^2}{2}}(\chi^2)^{\frac{(\underline{v}-2)}{2}}}{2^{\frac{\underline{v}}{2}}(\frac{\underline{v}-2}{2})!} \tag{4.18}$$

$$\chi^2 = \frac{(\underline{n}-1)\underline{s}^2}{\sigma^2} \tag{4.19}$$

$$\frac{(\underline{n}-1)\underline{s}^2}{\chi^2_{\underline{U};\underline{n}-1}} \preceq \sigma^2 \preceq \frac{(\underline{n}-1)\underline{s}^2}{\chi^2_{\underline{L};\underline{n}-1}} \tag{4.20}$$

$$\underline{P}(\underline{x}) = \frac{e^{-\mu}\mu^{-\underline{x}}}{\underline{x}!} \tag{4.21}$$

$$\underline{E}(\underline{x}) = \mu \tag{4.22}$$

$$\mathrm{Var}(\underline{x}) = \mu \tag{4.23}$$

$$\underline{v} = 2(\underline{r}+1) \tag{4.24}$$

$$\underline{v} = 2\underline{r} \qquad (4.25)$$

$$\underline{Z} = \frac{\underline{p} - \underline{p}_o}{\sqrt{\frac{\underline{p}_o(1-\underline{p}_o)}{\underline{n}}}} \qquad (4.26)$$

$$\underline{Z} = \frac{\underline{p}_1 - \underline{p}_2}{\underline{S}_{\underline{p}_1 - \underline{p}_2}} \qquad (4.27)$$

$$\underline{S}_{\underline{p}_1 - \underline{p}_2} = \sqrt{\hat{\underline{p}}(1-\hat{\underline{p}})(\frac{1}{\underline{n}_1} + \frac{1}{\underline{n}_2})} \qquad (4.28)$$

$$\hat{\underline{p}} = \frac{(\underline{n}_1)(\underline{p}_1) + (\underline{n}_2)(\underline{p}_2)}{\underline{n}_1 + \underline{n}_2} \qquad (4.29)$$

$$\underline{Z} = \frac{|\underline{Y}_1 - \underline{Y}_2| - .5}{\sqrt{\underline{Y}_1 + \underline{Y}_2}} \qquad (4.30)$$

$$\underline{Z} = \frac{\underline{n}_2\underline{Y}_1 - \underline{n}_1\underline{Y}_2}{\sqrt{(\underline{n}_1\underline{n}_2)}\ \sqrt{\underline{Y}_1 + \underline{Y}_2}} \qquad (4.31)$$

$$\underline{n} = \frac{[\,\underline{Z}_{\frac{\alpha}{2}} \sqrt{\underline{p}_2(1-\underline{p}_2)} + \underline{Z}_\beta \sqrt{\underline{p}_1(1-\underline{p}_1)}\,]^2}{\underline{d}^2} \tag{4.32}$$

$$\underline{n} = \frac{[\,\underline{Z}_{\alpha} \sqrt{\underline{p}_2(1-\underline{p}_2)} + \underline{Z}_\beta \sqrt{\underline{p}_1(1-\underline{p}_1)}\,]^2}{\underline{d}^2} \tag{4.33}$$

$$\chi^2 = \sum_{\underline{i}=1}^{\underline{k}} \frac{(\underline{O}_{\underline{i}} - \underline{E}_{\underline{i}})^2}{\underline{E}_{\underline{i}}} \tag{4.34}$$

$$\underline{R} = \frac{\sigma_{\underline{o}}}{\sigma_{\underline{H}}} \tag{4.35}$$

$$\underline{n} = 1 + \frac{1}{2}\left[\frac{\underline{Z}_{1-\alpha} + \underline{R}(\underline{Z}_{1-\beta})}{\underline{R}-1}\right]^2 \tag{4.36}$$

$$\underline{n} = 1 + \frac{1}{2}\left[\frac{\underline{Z}_{1-\alpha} + \underline{R}(\underline{Z}_{1-\beta})}{1-\underline{R}}\right]^2 \tag{4.37}$$

$$\text{Power} = \underline{P}\ [\underline{Z} < \frac{(LTL)-\underline{p}}{\sigma} \text{ or } \underline{Z} > \frac{(UTL)-\underline{p}}{\sigma}] \tag{4.38}$$

$$(P_a)(p) = AOQ \tag{4.39}$$

$$n\,\bar{p} - 2\sqrt{n\,p(1-\bar{p})} = 1 \tag{4.40}$$

$$\sqrt{n} = \frac{\sqrt{2-\bar{p}} + \sqrt{1-\bar{p}}}{\sqrt{\bar{p}}} \tag{4.41}$$

$$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = L \tag{4.42}$$

$$n = \frac{9\,\bar{p}(1-\bar{p})}{(L-\bar{p})^2} \tag{4.43}$$

$$n = Z^2_{\alpha/2}\frac{\bar{p}(1-\bar{p})}{d^2} \tag{4.44}$$

$$\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \tag{4.45}$$

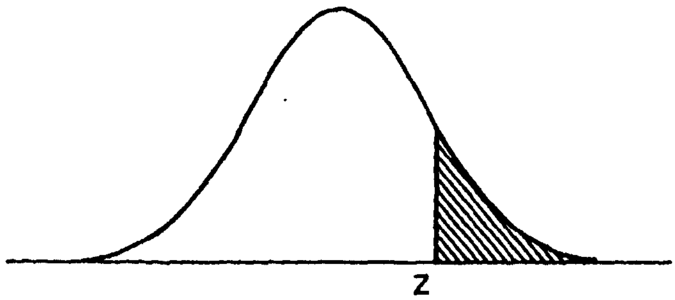
APPENDIX B

STATISTICAL TABLES

APPENDIX B-1

Normal Distribution

AREA BEYOND Z



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4	.3446	.3481	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

APPENDIX B-2

Distribution of Chi-Square

df	0.99	0.98	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	0.0157	0.0628	0.00393	0.0158	0.0642	0.148	0.455	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	0.0201	0.0404	0.103	0.211	0.446	0.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210	13.815
3	0.115	0.185	0.352	0.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.341	16.268
4	0.297	0.429	0.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277	18.465
5	0.554	0.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086	20.517
6	0.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812	22.457
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475	24.322
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090	26.125
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666	27.877
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209	29.588
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725	31.264
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217	32.909
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688	34.528
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141	36.123
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578	37.697
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.663	32.000	39.252
17	6.408	7.255	8.762	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409	40.790
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	25.869	32.346	34.805	42.312
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191	43.820
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566	45.315
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932	46.797
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289	48.268
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638	49.728
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980	51.179
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314	52.620
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642	54.052
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963	55.476
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278	56.893
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588	58.302
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892	59.703

APPENDIX C

DEFINITIONS OF TERMS

DEFINITIONS OF TERMS

Acceptable quality level (AQL). This is the limit of marginal quality or the worst acceptable average proportion tolerated by a sampling plan. This degree of quality (or better) is generally accepted at a confidence level given by 1-α.

Acceptance number ($\underline{c}$). Used in acceptance sampling; this value (or less) obtained in a sample is considered satisfactory.

Action limit. The point at which a decision is made to either accept or reject a hypothesized value is called the action limit.

Alpha (α). This value sets the significance level of a hypothesis test and signifies the probability that an incorrect decision has been made to reject the null hypothesis ($\underline{H}_0$).

Alternate hypothesis ($\underline{H}_A$). This is the hypothesis that the population parameter in question has a value different than that which is specified in the null hypothesis ($\underline{H}_0$).

Arithmetic mean. Also known as the average; it is determined by taking the sum of all the subsets in distribution and dividing it by the total value possible.

Average Outgoing Quality (AOQ). Because all processes posses some inherent degree of variability, some production runs will be of a better level of quality and others will be worse. Average outgoing quality represents the state of product conformance after scrap, sort and rework when it is submitted for shipment.

Average outgoing quality limit (AOQL). This represents the point where the average outgoing quality level will reach its greatest defect rate.

Beta (β). The probability that an incorrect decision has been made to accept the null hypothesis when it is, in reality, false is the beta risk. It is also called the type 2 error.

Binomial probability distribution. Also known as the Bernoulli distribution, it is based on the occurrence of one event or another event and gives the probability of the occurrence or nonoccurrence of the event in a given number of trials.

Collectively exhaustive. When the collection of all possible events are taken together to form the population, the events are said to be collectively exhaustive.

Combinations. A combination is a collection of subsets of $\underline{x}$ events taken from $\underline{n}$ events without regard to selection sequence.

Confidence interval. A specific range or interval of values which contains the probability (confidence level) that a specific value falls between, at a given level of risk (α), is known as a confidence interval.

Confidence limit. A range of values between which the population random variable is expected to be, at a specified probability level ($1-\alpha$), is contained by confidence limits.

Continuity correction factor. For a normal distribution approximation to the binomial random variable, a correction factor of .5 is used. While the normal distribution is a probability density function, the binomial is a probability mass function and is measured only at integer values. The continuity correction factor accounts for the undefined gaps in the binomial distribution.

Continuous variables. These variables possess the properties that they can assume any possible numeric value within a specific range or interval. For example, heights, weights or other quantitative measures are considered continuous because their values are bounded only by a certain predetermined degree of accuracy.

Control charts. Control charts are used in manufacturing to monitor (or improve) process performance and determine if a random variable is acting in a predictable manner.

Control limit. Action limits for control charting purposes are called control limits.

Critical value. A critical value is designated by $\underline{Z}_{\alpha}$ or $\underline{Z}_{\alpha/2}$ (see action limit).

Cumulative density. This represents the area of a continuous distribution, under a curve, between two given values.

Cycle. A cycle is a recurring movement of a specific sequence of events that generally tends to repeat itself in a reasonably consistent and measurable fashion.

Degrees of freedom ($\underline{df}$ or $\underline{v}$). When a population value is known or hypothesized, all values of possible scores are free to vary except for those which must be specifically defined in order to total up to the population value. The number which are allowed to vary are known as the degrees of freedom.

Deviation. A deviation is the difference between the population mean and another value in the distribution.

Discrete variables. These variables possess the properties that they can assume only certain values such as good, bad, 1, 2, etc., which have finite population measures and are often taken to be qualitative rather than quantitative in nature.

Expected frequency ($\underline{E}$). Given a null hypothesis about a population proportion, this value is the theoretically expected number associated with a certain class or category.

Hypothesis test. This is an experiment conducted to determine if a random variable is of a specific population distribution.

Lot tolerance percent defective (LTPD). A level of quality (percent defective) at a point which anything worse is considered unacceptable and has a small probability of being accepted during sampling is the LTPD of a sampling distribution. The probability of acceptance is generally set between nine and ten percent.

Independent events. Events are said to be independent when the occurrence of one event does not affect the probability of subsequent events.

Mean of binomial. The mean (μ) of the binomial distribution is denoted by $\underline{Np}$, where $\underline{p}$ represents the probability of occurrence and $\underline{N}$ represents the possible number of observations.

Mutually exclusive events. Two or more events are said to be mutually exclusive if they cannot both occur simultaneously.

$\underline{N}$ factorial ($\underline{N!}$). This is the product of each subsequent integer in a series from 1 to $\underline{N}$ by each preceding integer. For example, $4! = 4 \times 3 \times 2 \times 1$.

Normal probability distribution. This is a continuous probability distribution with an infinite number of possible values. It is bell-shaped and symmetrical. While it stands alone as a distribution, it is often used for approximations of discrete probability distributions.

Null hypothesis ($\underline{H}_0$). This is the hypothesis that a population parameter is of a specific value and is the measure at which a sample is compared (i.e., there is no detectable difference).

Observed frequency. The number of occurrences observed in a sample taken during an experiment or test gives the observed frequency.

Operational characteristic curve. This is a curve which indicates the probability of accepting a lot of material when the fraction defective is at a certain level and thus, describes the nature of a sampling plan.

One-tail test. In tests of hypothesis where only one extremity of a distribution is compared to the null hypothesis, the test is known as a one-tail test.

Out-of-control. The presence of factors other than those expected from the random variable of the distribution indicates a state of lack-of-control.

Parameter. A numeric measure which represents the true population characteristic such as the mean or standard deviation is known as a population parameter.

Permutation. A permutation is a collection of subsets of $\underline{x}$ events taken from $\underline{n}$ events, where order is important, and is the number of possible orders.

Population. The entire collection of events possible from a potential sample space is known as a population or sampling universe.

Power of a statistical test. The power is the strength of a hypothesis test in rejecting the null hypothesis ($\underline{H}_0$) when it is, in fact, false and is equal to $1-\beta$.

Probability density function. This describes the probability associated with a continuous random variable along a number line. Because the probability of any one event occurring is equal to zero, the probability is defined as falling between a range of values or within a specified area of the distribution. The sum of probability (or the area) along the continuum equals one.

Probability mass function. This describes the probabilities associated with a discrete random variable. The population random variable possesses a finite number of possible values with each having an assigned probability of occurrence. The sum of all the individual probabilities (or the mass) of the distribution equals one.

Random variable. This is a function which defines a sample space and the values contained within it.

Run. An unbroken sequence of directly analogous events that are preceded and followed by differing, random events is considered a run.

Sample. A sample is a subset of items taken from a population.

Sampling distribution. A theorized distribution based on statistical values such as $\underline{n}$ $\underline{p}$ or $\underline{s}/\sqrt{\underline{n}}$ is the sampling distribution of a population.

Sampling distribution of differences between means. A theorized distribution comprised of the differences of values between separate yet compared populations represents the distribution of differences between the means.

Significance level (α). This denotes a critical value or probability level for purposes of testing, helps define the risk of error and establishes a point at which decisions are based in tests of hypothesis.

Skewness. A distribution lacking symmetry and with more values on one side of the mean than the other is recognized as being skewed.

Standard deviation. The standard deviation is the square root of the variance.

Standard error of $\bar{\underline{p}}(\sigma_{\bar{p}})$. This is the standard deviation of the sampling distribution of $\bar{\underline{p}}$ values.

Standard error estimate of $\bar{\underline{p}}(\underline{s}_{\bar{p}})$. This is an estimate of $\sigma_{\bar{p}}$ from data taken during sampling; denoted by $\underline{s}/\sqrt{\underline{n}}$.

Statistic. A statistic is a numeric measure which is calculated from sample observations to describe population characteristics.

Trend. A trend represents the general long-term probability that a process gradually increases, decreases or remains the same.

Type-1 error. Also known as the alpha (α) error, this represents the probability of falsely rejecting the null hypothesis ($\underline{H}_0$) when it is true.

Type-2 error. Also known as the beta (β) error, this represents the probability of failing to reject the null hypothesis ($\underline{H}_0$) when it is false.

Two-tailed test. These are hypothesis tests where both the lower and upper extremities of the distribution are compared to the null hypothesis. This is contrasted with a one-tail test where the critical value is considered only on one side of the distribution.

Unimodal Curve. A unimodal curve is based on a probability distribution possessing only one point of highest frequency.

Universe. See population.

Variance. This is a measure of the variability of a random variable about a population (or sample) mean. It is obtained by taking the squared differences between the mean and all of the observed values and summing them together.

$\underline{Z}$-score. This is a measure based on standard deviation units obtained by determining the difference between the mean and a point on the distribution which is then divided by the standard deviation. The resulting value is called a $\underline{Z}$-score or standard score and is often used for testing or comparison purposes.

APPENDIX D

SYMBOL DEFINITIONS

SYMBOL DEFINITIONS

$\| \|$	=	absolute value
α	=	alpha: type-one error, significance level or the risk of rejecting the null hypothesis (H_0) when it is true.
β	=	beta: type-two error or the risk of accepting the null hypothesis (H_0) when it is, in reality, false.
c	=	the number defects expected from a sample taken from a Poisson distributed random variable. This value also represents the acceptance number associated with sampling plans.
C_x^n	=	the number of combinations possible when taking n things, x at a time.
d	=	a difference between means in a test of hypothesis: $d = p_1 - p_2$.
df or v	=	degrees of freedom.
e	=	a numeric constant (2.718+) commonly found in mathematics.
E	=	expected value or frequency.
$f(x)$	=	a function operator for the random variable x.
H_A	=	the alternate hypothesis.
H_0	=	the null hypothesis.
i	=	to denote or differentiate a quantity or variable in this text.
k	=	a subscript used to denote a quantity. Also used to represent the number of intervals or cells used in a chi-square goodness-of-fit test.

L	=	a limiting value or proportion; used in this text to establish action limits for control charts and for determining sample size selection.
LCL	=	lower control limit.
LTL	=	lower test limit.
$\underline{m}$	=	the number of defective units in a hypergeometric distributed population. Also used for the number of estimated parameters in a chi-square goodness-of-fit test.
μ	=	the population mean.
$\underline{n}$	=	the number of units taken or the size of the sample.
$\underline{n}\ \underline{p}$	=	population proportion for sampling purposes.
$\underline{N}$	=	the number of items in a population.
N!	=	$\underline{N}$ factorial; $\underline{N}$ ($\underline{N}$-1)($\underline{N}$-2)($\underline{N}$-3) ...1.
$\underline{O}$	=	the observed value found in the chi-square goodness-of-fit test.
$\underline{p}$	=	population proportion (or fraction) of occurrences.
$\underline{\bar{p}}$	=	the average proportion (or fraction) for the frequency of occurrences.
$\underline{\hat{p}}$	=	an estimator for $\underline{p}$.
$\underline{P}(\underline{A})$	=	the probability of the occurrence of event $\underline{A}$.
$\underline{P}_a$	=	probability of acceptance.
P_x^n	=	the number of permutations of $\underline{n}$ things taken $\underline{x}$ at a time.
$\underline{P}(\underline{x})$	=	probability of the occurrence of $\underline{x}$.

$\underline{v}$ = the number of occurrences in a sample.

$\underline{R}$ = a ratio.

$\underline{s}$ = sample standard deviation.
$\underline{s}^2$ = the sample variance.

$\underline{S}_{\underline{p}_1 - \underline{p}_2}$ = the standard error estimate of two population proportions, obtained by samples taken from populations $\underline{p}_1$ and $\underline{p}_2$

Σ = notation or operator for the summation process.

σ = population standard deviation.

σ^2 = population variance.

$\sigma_{\underline{p}}$ = standard error of $\underline{p}$.

$\sigma_{\underline{p}_1 - \underline{p}_2}$ = standard error of difference between two population proportions $\underline{p}_1$ and $\underline{p}_2$

UCL = upper control limit for control charting purposes.

UTL = upper test limit for tests of hypotheses.

v(x) = variance of the random variable $\underline{x}$.

$\underline{X}$ = a random variable.

$\underline{\bar{X}}$ = a sample average or mean.

χ^2 = chi-square random variable

$\underline{Y}$ = a symbol for count data taken in a test of hypothesis.

$\underline{Z}$ = a standard normal distribution value or $\underline{Z}$- score; $\underline{Z} = \frac{\underline{x} - \mu}{\sigma}$

APPENDIX E

INDEPENDENT RESEARCH SURVEY

INDEPENDENT RESEARCH SURVEY

1) Which of the following includes your experiences as an educator in teaching manufacturing process control ?

	Number of Responses yes	no
Corporate-base (in-house) training programs	3	0
Industrial Consultant	2	1
Seminars and/or Workshops	3	0
College or other academic institutions	2	1

2.a. As an instructor, have you used training devices as an aid to teaching discrete probability theory/sampling distributions or continuous approximations to discrete random variables ?

Number of Responses

yes 3 no 0

b. If no, do you intend to use (or may possibly use) such training devices in the future ?

Number Of Responses

yes - no -

3. If you answered yes to any portion of question 2, what types of applications have you found (or would find) training devices to be particularly useful in assisting the teaching of process control related topics ?

	Number of Responses	
	yes	no
Probability theory	3	0
Confidence interval estimations	1	1
Acceptance sampling	3	0
Sampling Variability	3	0
Control charting	3	0
Analysis of runs, trends or cycles	2	---
Pareto analysis	2	---
Operating characteristic principles	3	0
Power function	---	2
Hypothesis testing of means	2	1
Hypothesis testing of variances	2	1
Design and analysis of experiments	2	---
Distribution-free or non-parametric testing	1	1
Reliability or survival testing	1	1
Others	2	---

4. Of the training aids in common use, which of the following have you used or would consider using for teaching discrete probability theory/sampling distributions or continuous approximations to discrete random variables ?

	Number of Responses		
	have used;	would/will use;	would not use
Random Dice	2	1	---
Sampling Bowl	1	2	---
Sampling Box or Bead Box	3	---	---
Quincunx	2	---	---
Run Demonstrator	1	1	---
Roman Catapult	2	---	---

5. In order of importance to you as an instructor, please rank the following training devices for their overall, general use in teaching discrete probability/sampling distributions and continuous approximations to discrete random variables.
(5 = most important; 1 = least important)

	Priority Responses
Random Dice	5, 5, 3
Sampling Bowl	3, 4
Sampling Box	4, 5, 5
Quincunx	3, 5
Run Demonstrator	2, 3
Roman Catapult	1, 4

6. Please rank the following attributes associated with training devices in terms of priority by circling the appropriate number.
(5 = high priority; 1 = low priority)

	Priority Responses
Ease of use	5, 5, 5
Speed of sample selection	4, 5, 5
Easy sample counting	4, 4, 5
Efficient replacement of sample back to population	4, 4, 4
Maintenance of population parameters	5, 5, 5
Minimized sampling bias	4, 5, 5
Flexibility in sample size selection	4, 4, 4
Teaching merits of sample size selection	4, 5, 5
Light-weight construction	2, 2, 5
Portability	4, 5, 5
Compactness	4, 5, 5
Rigidity or durability	4, 5, 5
Other(s)	1, -, -

7. When considering the attributes listed below, which training device do you believe provides the best service; the sampling box or the sampling bowl ?

	Number of Responses		
	S. Box	S. Bowl	Same
Ease of use	3	0	0
Speed of sample selection	2	0	1
Easier sample counting	2	0	1
Efficient replacement of sample to back population	1	1	1
Maintenance of population parameters	1	1	1
Flexibility in sample size selection	0	2	1
Light-weight construction	2	1	0
Portability	3	0	0
Compactness	3	0	0
Rigidity or durability	2	0	1
Other (s)	0	0	1

8. Is it likely that the use of the Sampling Box will better improve the productivity of individual student participation in classroom related activities than does the Sampling Bowl?

Number Of Responses

yes 2 no 0 not sure 1

9. Is it possible that the Sampling Box will help to better clarify concepts which may be difficult for students to grasp than will the Sampling Bowl ?

Number Of Responses

yes 1 no 0 not sure 2

10. Do you believe that the student may learn more by using the Sampling Box than with the Sampling Bowl ?

Number Of Responses

yes 1 no 0 not sure 2

11. Is it possible that students will be able to more rapidly disseminate course content with the use of Sampling Box as opposed to the Sampling Bowl ?

Number Of Responses

yes 2 no 0 not sure 1

12. Is it probable that the use of the Sampling Box minimizes the potential for classroom disruption better than does the Sampling Bowl?

Number Of Responses

yes 1 no 1 not sure 1

13. Do you believe that the use of the Sampling Box will help to foster better student commitment to the principles you promote than expected when using the Sampling Bowl ?

Number Of Responses

yes 2 no 0 not sure 1

14. Considering your ideal classroom setting; would you prefer to use the Sampling Box, Sampling Bowl or the use of both in conjunction with teaching of manufacturing process control concepts to your students ?

Number Of Responses

S. Box 3 S. Bowl 0 both in conjunction 0

APPENDIX F

EVALUATION

3/21/1995
Schorr Training and Consulting
[redacted]
South Beloit, Il. 61080

Dear Vivienne Schorr
I am a graduate student in the Industrial Education and Technology Department at Western Illinois University currently conducting research for my Masters' thesis and development of the final manuscripts. The topic I have chosen is "The use of Sampling Device as an Aid to Teaching Manufacturing Process Control."

In order for me to adequately address this topic, I am in need of your forthright appraisal and recommendations in responding to the enclosed questionair relating to the supplied partial manuscripts and sampling device. Completion of the questionnaire should take no more than thirty minutes. Although the enclosed partial manuscripts may seem to be a prolix of information, they are supplied merely for the purpose of exhibiting the type of logic used to develop the sampling device and the class of applications it is intended to help parallel in classroom settings. Your developmental advice will prove to be an invaluable source in determining the relative merits of a proposed Sampling Box as compared to other training devices and establishing the nature of classroom applications the Sampling Box would be well suited to stanchion.

You are one of only very few independent resources selected because of your expertise and integrity. Your responses will maintain complete anonymity (even to me) throughout the duration of this research and will not be opened until all responses have been returned. Please feel free to complete the enclosed questionair and seal it in the supplied, unmarked envelope for later review.

Your candor and support will be greatly appreciated.
Sincerely,

Jeffrey P. Bunger
Enclosures 4

3/15/1995
Roy C. Ervin
Sundstrand Aerospace Corp.
[redacted]
Rockford, Il 61125

Dear Roy C. Ervin
I am a graduate student in the Industrial Education and Technology Department at Western Illinois University currently conducting research for my Masters' thesis and development of the final manuscripts. The topic I have chosen is "The use of Sampling Device as an Aid to Teaching Manufacturing Process Control."

In order for me to adequately address this topic, I am in need of your forthright appraisal and recommendations in responding to the enclosed questionair relating to the supplied partial manuscripts and sampling device. Completion of the questionnaire should take no more than thirty minutes. Although the enclosed partial manuscripts may seem to be a prolix of information, they are supplied merely for the purpose of exhibiting the type of logic used to develop the sampling device and the class of applications it is intended to help parallel in classroom settings. Your developmental advice will prove to be an invaluable source in determining the relative merits of a proposed Sampling Box as compared to other training devices and establishing the nature of classroom applications the Sampling Box would be well suited to stanchion.

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Your candor and support will be greatly appreciated.
Sincerely,

Jeffrey P. Bunger
Enclosures 4

3/15/1995
Robert A. Dovich
[redacted]
Rockford, Il. 61108

Dear Robert A. Dovich
I am a graduate student in the Industrial Education and Technology Department at Western Illinois University currently conducting research for my Masters' thesis and development of the final manuscripts. The topic I have chosen is "The use of Sampling Device as an Aid to Teaching Manufacturing Process Control."

In order for me to adequately address this topic, I am in need of your forthright appraisal and recommendations in responding to the enclosed questionair relating to the supplied partial manuscripts and sampling device. Completion of the questionnaire should take no more than thirty minutes. Although the enclosed partial manuscripts may seem to be a prolix of information, they are supplied merely for the purpose of exhibiting the type of logic used to develop the sampling device and the class of applications it is intended to help parallel in classroom settings. Your developmental advice will prove to be an invaluable source in determining the relative merits of a proposed Sampling Box as compared to other training devices and establishing the nature of classroom applications the Sampling Box would be well suited to stanchion.

You are one of only very few independent resources selected because of your expertise and integrity. Your responses will maintain complete anonymity (even to me) throughout the duration of this research and will not be opened until all responses have been returned. Please feel free to complete the enclosed questionair and seal it in the supplied, unmarked envelope for later review.

Your candor and support will be greatly appreciated.
Sincerely,

Jeffrey P. Bunger
Enclosures 4

EVALUATION

This evaluation has been generated for the purpose of assessing the intrinsic strength and weaknesses of the thesis. Please read and consider the following question and mark each entry as objectively as possible.

Please rate the performance of this researcher based on the following classifications:

1. poor
2. below average
3. average
4. above average
5. excellent

A. Did the organization of this paper follow a logical pattern of development?

1 2 3 (4) 5

B. Were the ideas and concepts explored in this paper consistent with the aims of the researcher?

1 2 3 4 (5)

C. Did the author readily convey key points and issues in a comprehendable manner?

1 2 3 (4) 5

D. Did the author satisfactorily develop each concept presented in this paper ?

1 2 3 (4) 5

E. Was the nature and volume of technical material in this paper thoroughly developed and defined?

1 2 3 4 (5)

F. Does the author appear to have adequately researched this topic and command an acceptable mastery over the concepts presented in this paper ?

1 2 3 4 (5)

OVERALL THE WORK WAS VERY GOOD AND AS GOOD AS ANY WORK OF THIS TYPE I HAVE SEEN.

EVALUATION

This evaluation has been generated for the purpose of assessing the intrinsic strength and weaknesses of the thesis. Please read and consider the following question and mark each entry as objectively as possible.

Please rate the performance of this researcher based on the following classifications:

1. poor
2. below average
3. average
4. above average
5. excellent

A. Did the organization of this paper follow a logical pattern of development?

1 2 3 4 (5)

B. Were the ideas and concepts explored in this paper consistent with the aims of the researcher?

1 2 3 (4) 5

C. Did the author readily convey key points and issues in a comprehendable manner?

1 2 3 4 (5)

D. Did the author satisfactorily develop each concept presented in this paper ?

1 2 3 (4) 5

E. Was the nature and volume of technical material in this paper thoroughly developed and defined?

1 2 3 4 (5)

F. Does the author appear to have adequately researched this topic and command an acceptable mastery over the concepts presented in this paper ?

1 2 3 4 5

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Please rate the performance of this researcher based on the following classifications:

1. poor
2. below average
3. average
4. above average
5. excellent

A. Did the organization of this paper follow a logical pattern of development?

1 2 3 (4) 5

B. Were the ideas and concepts explored in this paper consistent with the aims of the researcher?

1 2 3 4 (5)

C. Did the author readily convey key points and issues in a comprehendable manner?

1 2 3 (4) 5

D. Did the author satisfactorily develop each concept presented in this paper ?

1 2 3 4 (5)

E. Was the nature and volume of technical material in this paper thoroughly developed and defined?

1 2 3 4 (5)

F. Does the author appear to have adequately researched this topic and command an acceptable mastery over the concepts presented in this paper ?

1 2 3 4 5

APPENDIX G

IE&T 592 THESIS PRESENTATIONS ATTENDED

IE&T 592 THESIS PRESENTATION ATTENDED

Bright, George
(Patent Process)

Del Valle, Angelica
(Drive Torque Performance Fine Threads)

Hansell, Robert W.
(Resistance Welding)

Park, Sang-Jin
(Material Flow)

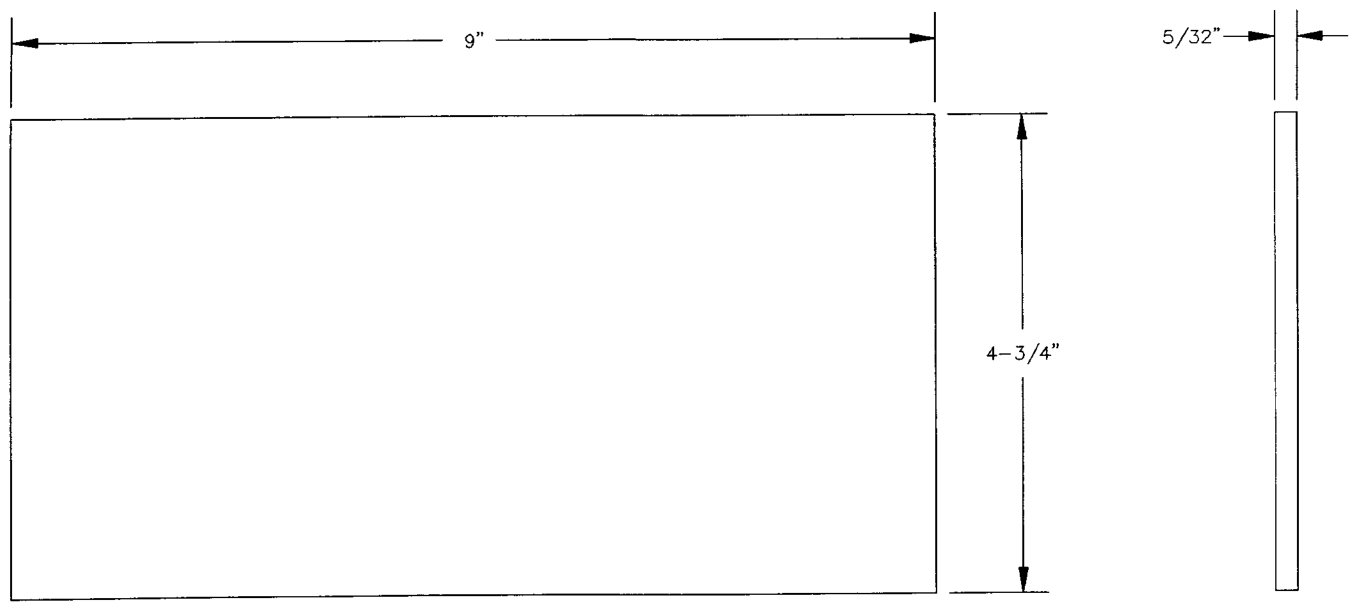
Patton, Gay
(John Deere Technical Survey)

Sharp, Steve
(Robotics)

Snowden, Shawn
(Measurement)

APPENDIX H

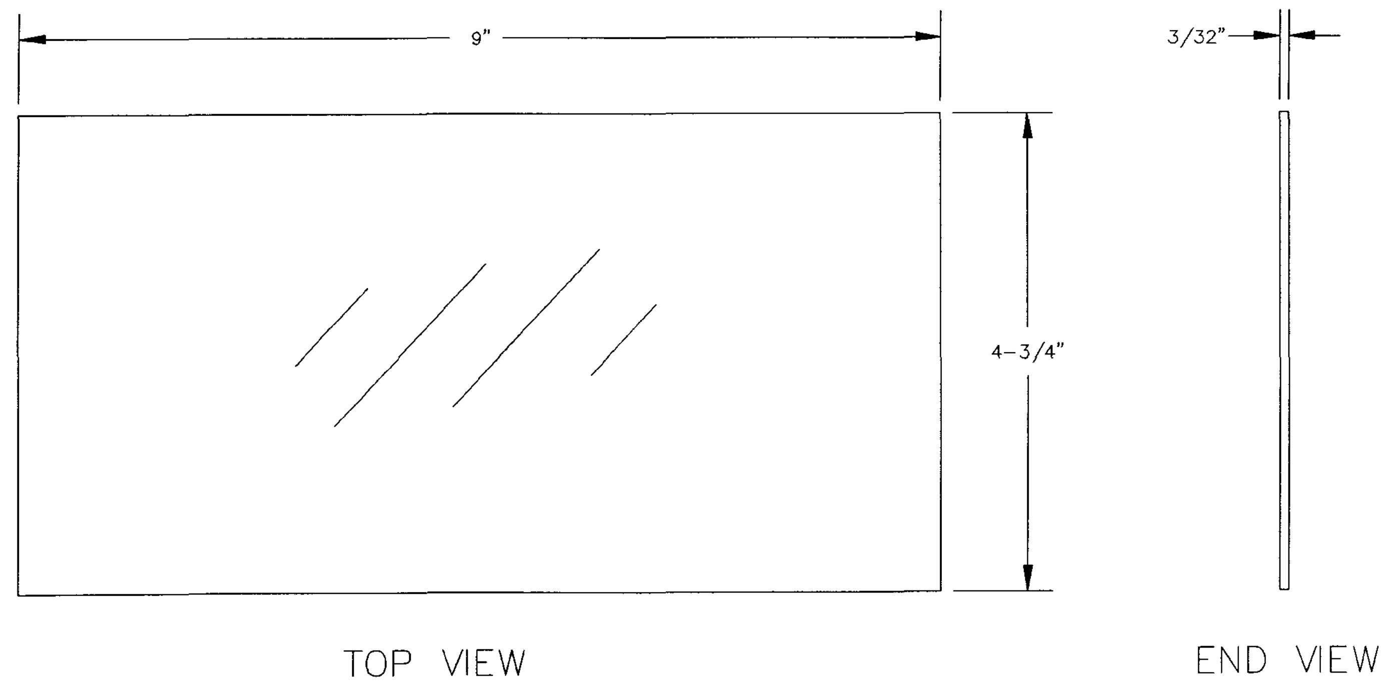
ENGINEERING SPECIFICATIONS



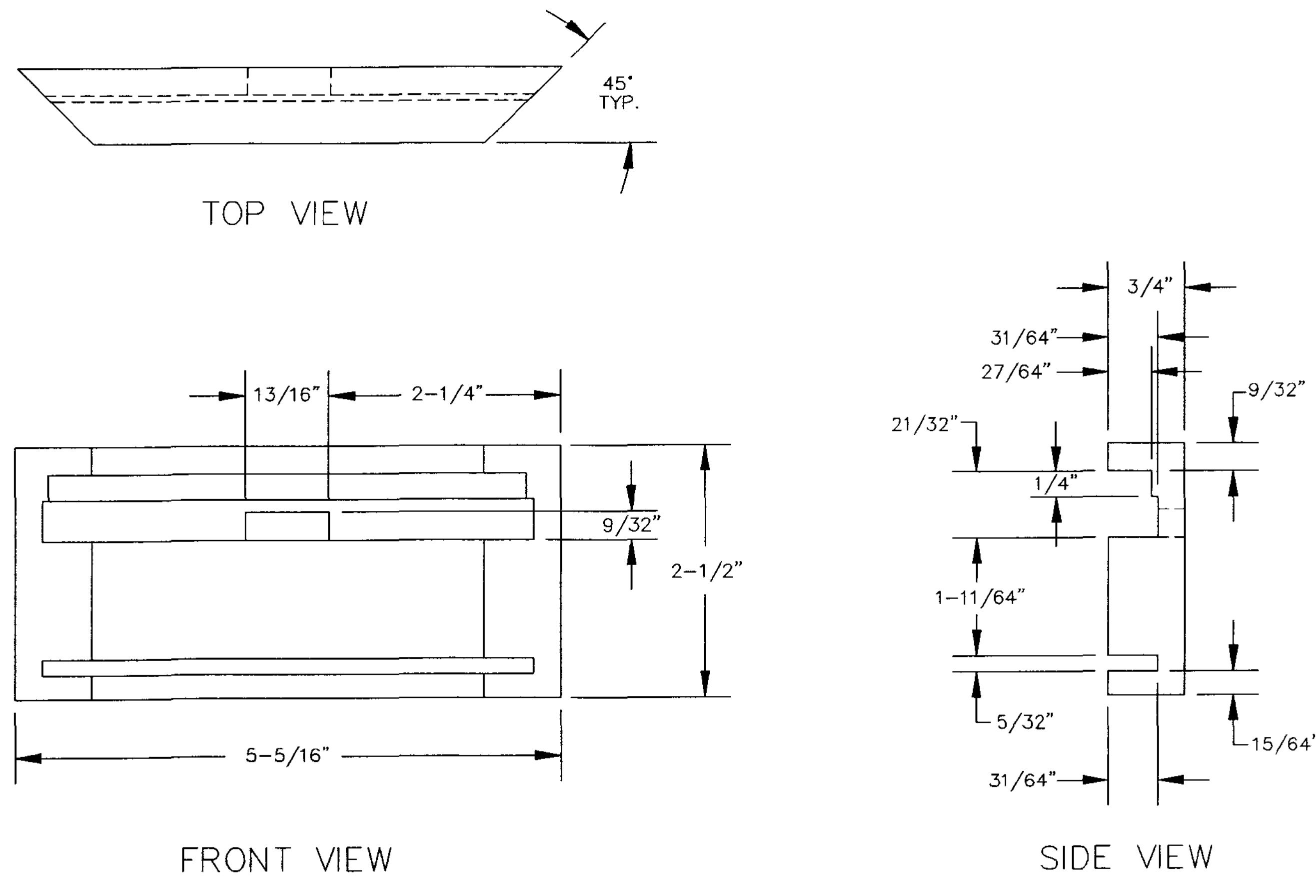
TOP VIEW

END VIEW

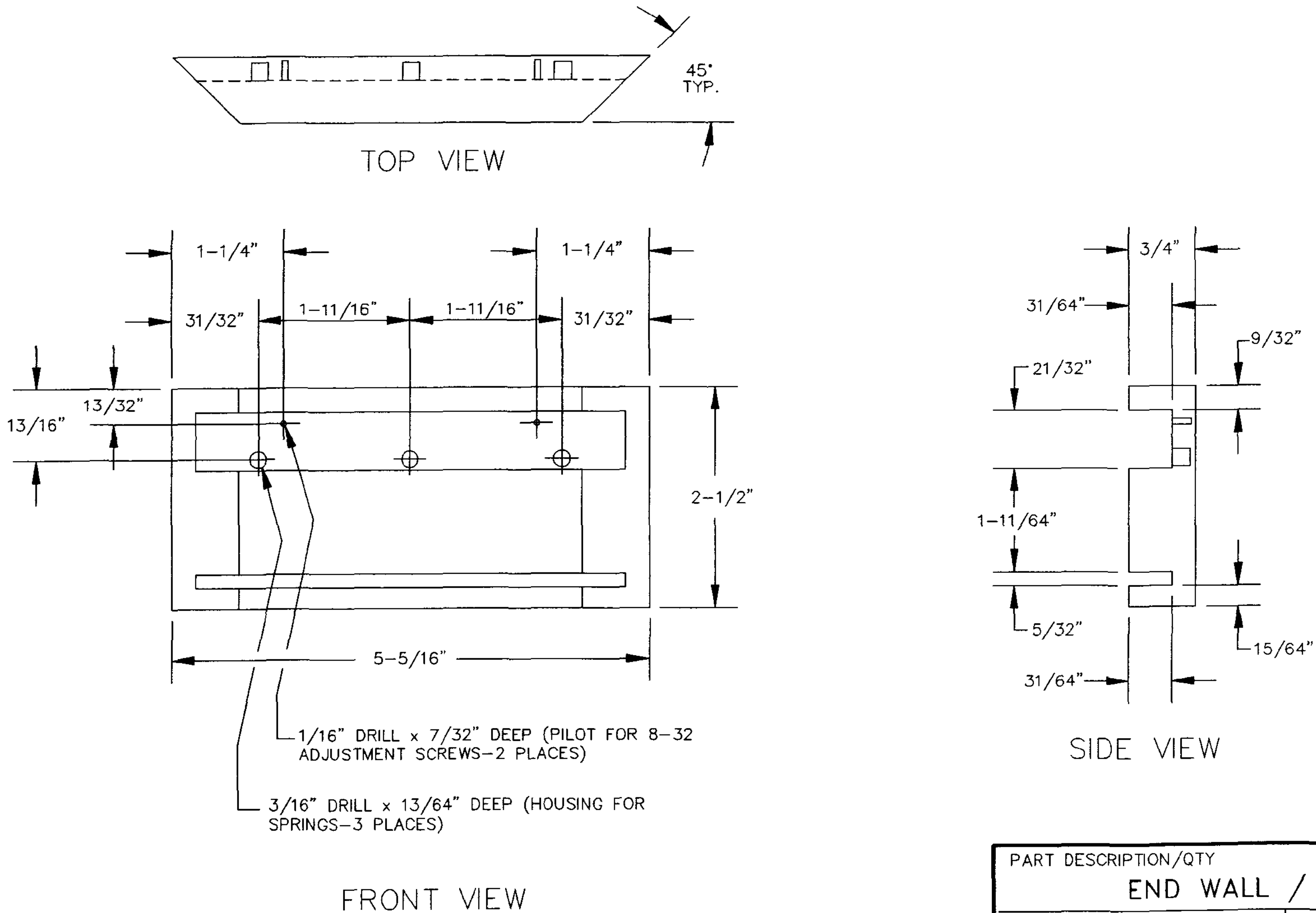
PART DESCRIPTION/QTY	
FLOOR / 1	
SCALE 1/2	MATERIAL PINE
DRAWN BY J. BUNGER	DATE 8/23/93



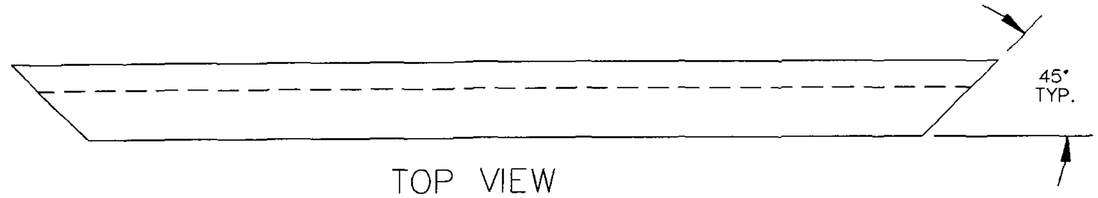
PART DESCRIPTION/QTY TOP VIEWING PLATE / 1	
SCALE 1/2	MATERIAL PLEXIGLASS
DRAWN BY J. BUNGER	DATE 8/23/93



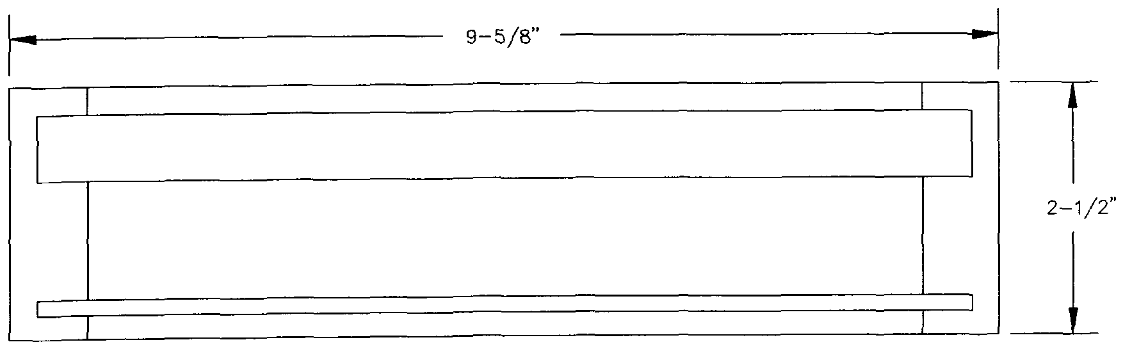
PART DESCRIPTION/QTY	
LEVER END WALL / 1	
SCALE 1/2	MATERIAL PINE
DRAWN BY J. BUNGER	DATE 8/23/93



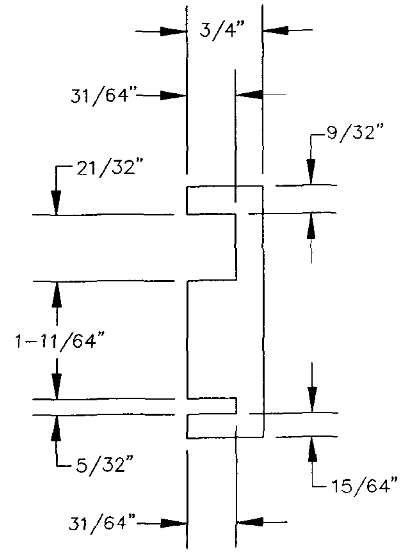
PART DESCRIPTION/QTY	
END WALL / 1	
SCALE 1/2	MATERIAL PINE
DRAWN BY J. BUNGER	DATE 8/23/93



TOP VIEW

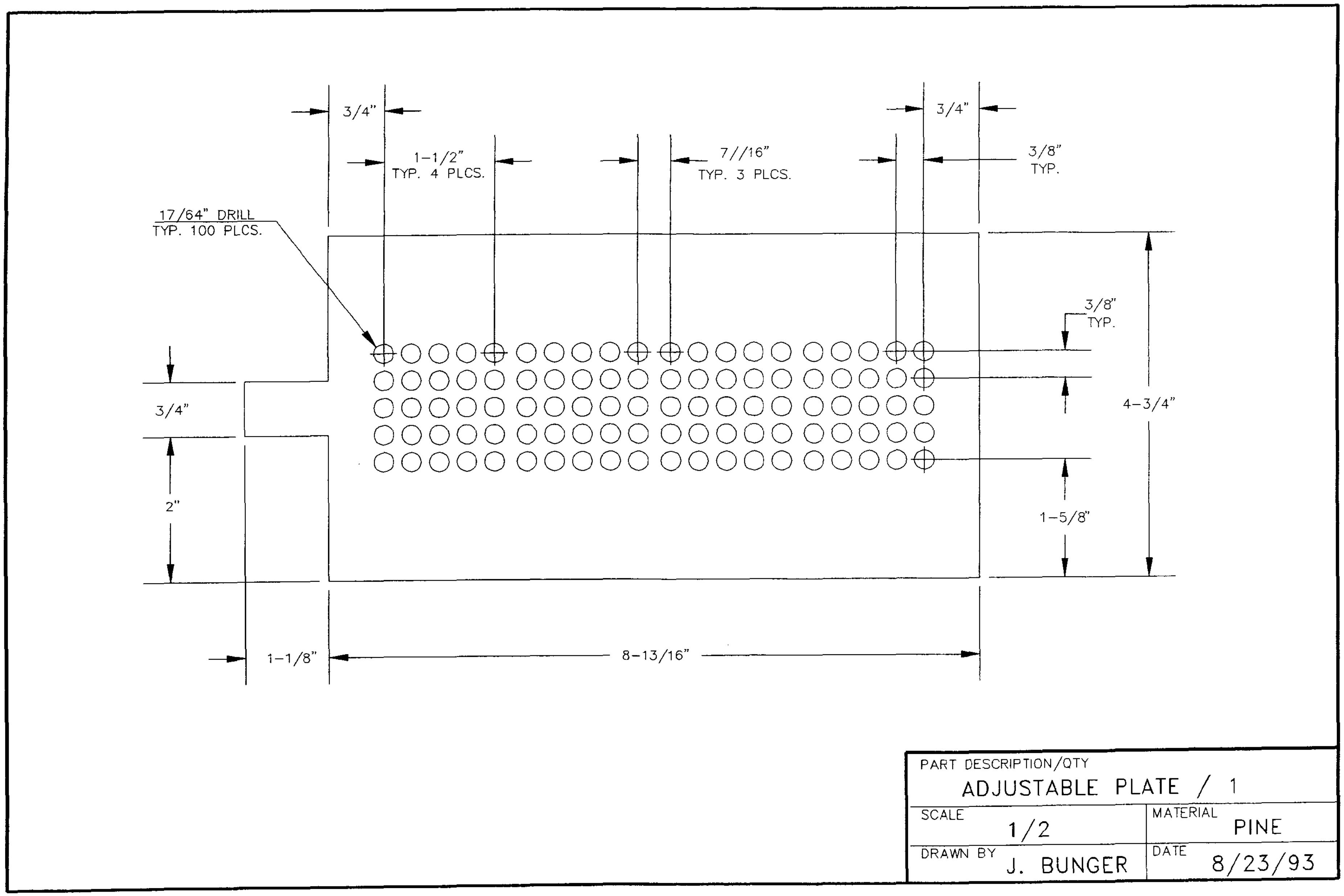


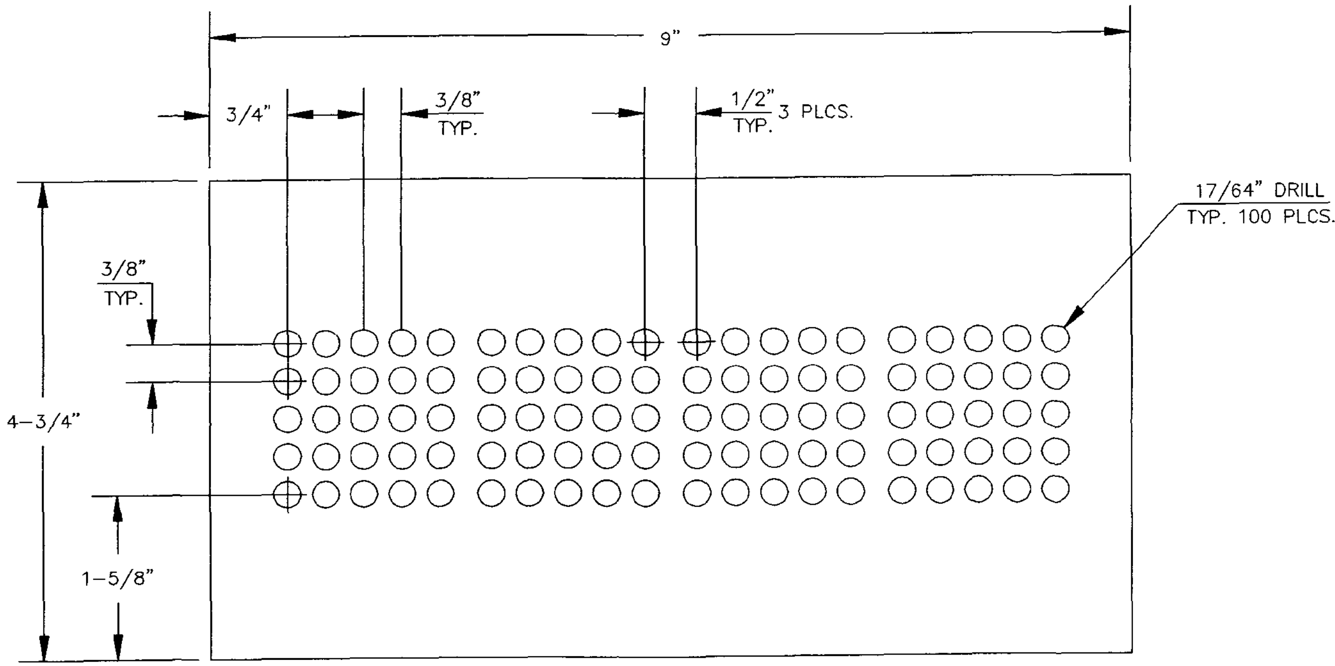
FRONT VIEW



SIDE VIEW

PART DESCRIPTION/QTY	
FRONT & BACK WALL / 2	
SCALE 1/2	MATERIAL PINE
DRAWN BY J. BUNGER	DATE 8/23/93


3/4"
3/4"
1-1/2"
TYP. 4 PLCS.
7//16"
TYP. 3 PLCS.
3/8"
TYP.
17/64" DRILL
TYP. 100 PLCS.
3/8"
TYP.
3/4"
4-3/4"
2"
1-5/8"
1-1/8"
8-13/16"
PART DESCRIPTION/QTY
ADJUSTABLE PLATE / 1
SCALE
1/2
MATERIAL
PINE
DRAWN BY
J. BUNGER
DATE
8/23/93



PART DESCRIPTION/QTY
STATIONARY PLATE / 1

SCALE 1/2	MATERIAL ALUMINUM
DRAWN BY J. BUNGER	DATE 8/23/93

APPENDIX I

LETTERS OF PERMISSION

3/28/1995

To: ASQC Quality Press
Acquisition Dept
[redacted]
P.O.Box [redacted]
Milwaukee, WI 53201-[redacted]

From: Jeffrey P.Bunger
Graduate Student
Western Illinois University
[redacted], Apt. [redacted]
Addison, Il. 60101
([redacted]) [redacted]-[redacted]

I, Jeffrey P. Bunger, hereby request permission to reprint the following material:
Dovich, Robert A.(1992).Quality Engineering Statistics. Milwaukee, WI: ASQC Quality Press. Table A(cont.), Normal Distribution, Area Beyond Z.p.94

The material will appear in the following volume:
Bunger, Jeffrey P. (1995). The Use of a Sampling Device as an Aid to Teaching Manufacturing Process Control.
Publisher: Western Illinois University
Probable publication date: May, 1995.
Form of publication: Thesis. Approximate number of pages: 200.

Please indicate permission as granted below and return the original copy of this letter. A duplicate copy is enclosed for your files.

Thank you.

Sincerely,

[redacted]
Jeffrey P. Bunger

[redacted] 3-30-95
(Permission granted) (Date)

Credit line if a special one is needed.

Please note the book, author & publisher

3/28/1995

To: James Warren
President
Lightening Calculator
P.O.Box [redacted]
Troy, Michigan 48099-[redacted]
([redacted]) [redacted]-[redacted]

From : Jeffrey P. Bunger
Graduate Student
Western Illinois University
[redacted] Apt. [redacted]
Addison, Il. 60101
([redacted]) [redacted]-[redacted]

I, Jeffrey P. Bunger, hereby request permission to reprint the following material:
Warren, James. (CEO). Promotional Literature and Video Cassete Tape. Lightening Calculator: Troy Michigan. Illustration of Quincunx and Video Cassette Tape.

The material will appear in the following volume:
Bunger, Jeffrey P. (1995). The Use of a Sampling Device as an Aid to Teaching Manufacturing Process Control.
Publisher: Western Illinois University
Probable publication date: May, 1995.
Form of publication: Thesis. Approximate number of pages: 200.

Please indicate permission as granted below and return the original copy of this letter. A duplicate copy is enclosed for your files.

Thank you.

Sincerely,

[redacted]

Jeffrey P. Bunger

[redacted]

(Permission granted)

4/4/95

(Date)

Credit line if a special one is needed.

IE&T 592 THESIS PRESENTATION ATTENDANCE SHEET

THE USE OF A SAMPLING DEVICE AS AN AID TO TEACHING MANUFACTURING PROCESS CONTROL

Presented by

Jeffrey P. Bunger

on

April 28, 1995

[redacted]